PRICES AND FREQUENCIES OF PUBLIC TRANSPORT WITH AND WITHOUT OPTIMAL CAR TRAVEL PRICING: EVIDENCE FROM SWEDEN

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1 INTRODUCTION

It is a generally held view by economists that car driving is sub-optimally priced in urban areas. Irrespective of whether politicians are aware of the sub-optimal pricing, one may interpret that they understand the second-best rule for pricing of public transport, based on the fact that urban public transport in most industrialised countries is partly financed by its users and partly by taxes.

Short-comings of second-best pricing, besides non-optimal levels of congestion, air pollution etc., are that the public transport service worsens and excess burden becomes larger than what would be the case with first-best pricing. Since motorists are generally relatively insensitive to public transport fares, the only way to reduce congestion, improve the environment and increase the standard of public transport seems to be road user charges.

Section 2 of this paper includes a somewhat shortened version of paper 4 in Jansson (1991). Section 3 includes a computer simulation of public transport prices and service frequencies as a response to a move from non-optimal to optimal car travel pricing in the Stockholm region.

2 MODEL ANALYSIS

2.1. Introduction

In the absence of road pricing a "second-best policy" where the public transport fare is set below its marginal cost has often been discussed (see e. g. Glaister [1974]). An issue that does not seem to have been analysed in the literature is whether bus passengers are better off with or without road pricing. If this were known, it could have an influence on the transport policy. In this paper we examine the implications of superimposing road-pricing not only on an adopted second-best policy, but also on two adopted alternative policies. The implications are discussed in terms of price for public transport as well as service frequency. One policy is the second-best policy hinted at above. Another policy is a "bus optimum policy", which assumes that a welfare maximising authority is dealing with bus service only, but where the operator and its passengers are negatively affected by car traffic. A third policy discussed is a "rule of thumb policy", which is widely used in practice, both by publicly and privately run bus operators. This is a policy not based on optimisation, but where the operator has to cover a certain percentage of costs (> or < 100%) and keep a fixed load factor. With the second-best policy and the "bus optimum policy" it is found that introduction of road pricing would increase both optimal price and optimal service frequency, but with the "bus optimum policy" frequency would be increased relatively more and price relatively less. On the other hand, with the "rule of thumb policy", which we believe is the most common policy, public transport passengers would definitely be better off with than without road pricing, since optimal price would be lower and optimal frequency higher.

The paper follows the tradition in which price and frequency are optimised simultaneously (c.f.Mohring [1972], Turvey and Mohring [1975], J. O. Jansson [1979],[1984] and Panzar [1979]). The model presented in paper 1 is here extended in order to deal simultaneously with bus and car pricing as well as bus service frequency.

The institutional framework and basic assumptions are presented in section 2.2. In section 2.3 maximisation of a welfare function provides the basic optimality conditions for both the first-best and the two optimisation policies. In section 2.4 the implications of road pricing for each of three alternative policies are analysed. Section 2.5 states the conclusions.

2.2 Institutional framework and basic assumptions

A Transport Authority is in charge of public transport prices and service frequencies. The authority, or a public body working with it, may also be in charge of car travel pricing.

There is a group of people, all of whom have the option to use either bus or car on a road from the outskirts of a city to the Central Business District. The reason for dealing with bus specifically is that buses, as cars, cause environmental and congestion costs. Public transport modes other than bus may be analysed analogously, bearing in mind that certain considerations, specifically competition for road space between bus and car, would not appear for other public transport modes than bus. Main results are valid, however, for all urban public transport modes.

We use indices to identify modes of transport, thus letting 1 denote private transport and 3 denote bus transport. Mode specific prices, p_1 and p_3, are indicated by subscript, while all other variables and parameters related to mode are indicated by superscript. Throughout arguments of functions are delimited by [], while polynoms are delimited by ().

Optimality is achieved by maximisation of welfare, defined as consumers' plus producer's surplus minus environmental costs, where the latter are e.g. air pollution, noise and external accident costs. The analysis refers to one period, e.g., the morning peak, but can be repeated for any period, assuming interdependencies between periods are negligible. The remainder of this section defines transport costs and passengers' preferences, demand and costs.

Transport costs

The bus route is a γ kms round trip. The distance travelled by the passengers we consider is also denoted γ, bearing in mind that it is assumed in fact to constitute a fixed part of γ. The round-trip time of the service is $bq^3+\gamma r^3[X^1, F]$,where b is fixed boarding time per passenger, X^3 is the number of passengers in the time period, q^3 ($\equiv X^3/F$) is the number of bus passengers per departure, r^i is the run time per kilometre that is independent of number of passengers but dependent on congestion, i.e., on the hourly flow of cars and buses. Frequency, F, is the number of departures per hour. The number of vehicles needed is $F(bq^3+\gamma r^3[X^1, F])$. If $C^3[\ ]$ denotes the cost per departure, the total operating costs are thus:

$$(1) \qquad FC^3[q^3, X^1, F] \equiv F(c(bq^3+\gamma r^3[X^1, F]) + c^{\gamma}\gamma) \equiv F(Bq^3+O[X^1, F])$$

where c is the hourly capital, distance and labour cost per departure. Where there is no need for detailed specification, the denotation to the right is used. B ($\equiv\partial C^3/\partial q^3\equiv$ cb) is the operator's cost for a marginal boarding passenger (assumed to be constant due to constant boarding time, b) and O is operation-dependent time and distance cost.

Each car and bus is assumed to give rise to external environmental costs, $E^1[X^1,F] \equiv \gamma e^1[r^1[X^1, F]]$ and $E^3[X^1,F] \equiv \gamma e^3[r^3[X^1, F]]$, respectively, where e^i is the environmental cost per kilometre and $\partial E^i/\partial X^1$ and $\partial E^i/\partial F > 0$ (i=1,3).

Passenger preferences, demand and costs

The aggregate consumers' surplus, of a homogenous group of travellers, choosing between car and bus, is expressed as a function of the "generalised cost", $G = p + \phi\varphi$, where φ is the vector of the travel time components and ϕ is the vector of monetary time values, assumed to be the same for all individuals, i.e., the same for all at each point $[p,\varphi]$, but where ϕ may be a function of φ. The vector φ is here comprised of riding time and "frequency delay" for public transport and of riding time for car transport.

Riding time cost for bus passengers, T^3, is the product of the riding time and the value of riding time, ϕ^3, assumed to be dependent on occupancy rate, $R \equiv q^3/\sigma$, where σ is the number of seats, i.e., the more crowded the vehicle is, the more onerous riding becomes per minute.

(2) $$T^3 \equiv \phi^3[q^3/\sigma](bq^3+\gamma r^3[X^1, F3])$$

Note that riding time cost varies positively with frequency due to worse congestion, via r^3, but negatively due to a lower occupancy rate, i.e., less crowding, via ϕ^3.

The motorists' riding time also depends on road congestion but the value of riding time, ϕ^1 , is assumed to be constant, since there is no correspondence to crowding in buses. The riding time cost is then:

(3) $$T^1 \equiv \phi^1\gamma r^1[X^1, F]$$

The interval between bus departures is $1/F$. Ideal departure time, t, is in the interval $0 \leq t \leq 1/F$. Frequency delay, $1/F - t$, is defined as the time interval between the actual departure time and the passenger's ideal departure time. The cost of frequency delay for passengers with the ideal departure time t is $T^\tau[F,t] \equiv \phi^\tau[1/F - t](1/F - t)$, where the value of frequency delay, ϕ^τ, may be assumed to vary with the delay.

Since frequency delay cost is part of generalised cost and has a specific value for each ideal departure time, t, demand may also depend on t, so that $x^3[t]$, is demand at time t in the interval $0 \leq t \leq 1/F$. The three concepts of demand for public transport are thus related in the following manner:

(4) $$X^3 \equiv Fq^3 \equiv F\int_0^{1/F} x^3[t]dt$$

Evidently $\partial X^3/\partial F > 0$, and we assume throughout that $\partial q^3/\partial F < 0$, i.e., that frequency elasticity is less than unity. Otherwise a doubling of frequency would for instance generate more than the double number of passengers, which is unlikely. Demand for each mode is assumed to be a function of generalised cost for both modes, i.e., $X^i = f^i[G^1,G^3]$. It is assumed that variations in our policy variables, prices and frequency, affect the total number of journeys to a negligible extent, so that such variations lead to travellers shifting mode, while total demand, X^1+X^3, is constant.

The definitions of riding time and frequency delay costs imply the following expressions for generalised cost for the individual in each group:

(5a) $$G^1 \equiv p_1 + T^1[X^1[\mathbf{p}, F], F]$$

(5b) $$G^3 \equiv p_3 + T^3[q^3[\mathbf{p}, F], X^1, F] + T^\tau[F,t]$$

where **p** is vector notation of the two prices and where generalised cost for public transport, G^3, refers to the ideal departure time t.

The reservation price in generalised cost terms for the individual with the maximum reservation price is called G^{max}. The consumers' surplus for passengers having the ideal departure time t is denoted $s^i[G^i]$. The total consumers' surplus, $S^i[G^i]$, at actual $G^i = p_i + \phi^i s^i$ is then:

$$(6a) \qquad S^1[G^1] \equiv \int_{G^1}^{G^{max}} X^i[\rho]d\rho$$

$$(6b) \qquad S^3[G^3] \equiv F \int_0^{1/F} s^3[G^3[t]]dt \equiv F \int_0^{1/F} \int_{G^3}^{G^{max}} (x^3[\rho]d\rho)dt$$

The relations between demand, generalised cost and surplus are:

$$(7a) \qquad \partial S^1/\partial G^1 \equiv -X^1$$

$$(7b) \qquad \partial S^3/\partial G^3 \equiv -F\int_0^{1/F} x^3[G^3]dt \equiv -Fq^3 \equiv -X^3$$

2.2 Optimisation of prices and frequencies

We will here describe the general conditions from which we achieve optima for three policies: "first-best policy", "second-best policy" and "bus optimum policy". We maximise a welfare function, w, comprised of the sum of consumers' and producer's surplus minus environmental costs, with respect to the three policy variables, i.e., the two prices and the frequency:

$$(8) \qquad w[\Sigma_i S^i[G^i[\mathbf{p},F]] + \Sigma_i \pi^i - \Sigma_i E^i] \equiv$$

$$\equiv S^3[G^3[\mathbf{p},F,t]] + p_3 X^3 - FC^3[q^3,X^1,F] - FE^3[X^1,F] +$$

$$+ S^1[G^1[\mathbf{p},F]] + p_1 X^1 - X^1 E^1[X^1,F]$$

The first-order conditions with respect to the prices p_i (i=1,3) are derived in the appendix and are:

$$(9) \qquad w_i = X_i^1 (p_1 - X^1 T_1^1 - X^3 T_1^3 - FC_1^3 - X^1 E_1^1 - E^1 - FE_1^3) +$$
$$+ X_i^3 (p_3 - X^3 T_3^3 - FC_3^3) \equiv \Sigma_j X_i^j (p_j - m_j) = 0$$

where subscripts are used for partial derivatives, so that e.g. $\frac{\partial w}{\partial p_1} \equiv w_1$ and $\frac{\partial T^3}{\partial X^1} \equiv T_1^3$, and where m_j (j=1,3) denote social marginal costs. By using the specifications of functions (1), (2) and (3), marginal costs can be expressed in more detail as:

$$(10a)\qquad m_1 \equiv X^1\,\gamma\phi^1\frac{\partial r^1}{\partial X^1} + X^3\,\gamma\phi^3\frac{\partial r^3}{\partial X^1} + F\gamma\frac{\partial r^3}{\partial X^1} + X^1\,\gamma\frac{\partial e^1}{\partial r^1}\frac{\partial r^1}{\partial X^1} + F\gamma\frac{\partial e^3}{\partial r^3}\frac{\partial r^3}{\partial X^1}$$

$$(10b)\qquad m_3 \equiv X^3\left(\frac{\partial \phi^3}{\partial R^3}\frac{1}{F\sigma}\left(b\frac{X^3}{F} + \gamma r^3\right) + \phi^3\frac{b}{F}\right) + bc$$

The costs caused by an additional motorist include, in addition to the effects on other cars, negative external effects borne by the bus passengers, the public transport operator and the environment. Marginal costs caused by an additional bus passenger are borne by other bus passengers and the operator, but do not explicitly include external costs borne by motorists and environment, since frequency is kept constant in the partial derivation with respect to price. Note also that marginal cost and optimal price of a car trip are directly proportional to distance travelled. For a bus trip, however, only the part of marginal cost that reflects crowding is directly dependent on distance travelled.

By denoting the difference between price and marginal cost $d_i \equiv p_i - m_i$ (i=1,2),we can write (9) in matrix form as:

$$(11)\qquad \begin{pmatrix} X_1^1 & X_1^3 \\ X_3^1 & X_3^3 \end{pmatrix}\begin{pmatrix} d_1 \\ d_3 \end{pmatrix} = 0$$

where, if all policy variables are available, $d_i \equiv p_i - m_i = 0$ corresponds to first-best optimum.

Our assumption that total demand is constant implies that the first-order condition with respect to frequency can be written as follows both for first-best and non-first-best situations (see appendix):

$$(12)\qquad \frac{\partial w}{\partial F} = m_3\frac{X^3}{F} - C - F\frac{\partial O}{\partial F} - X^3\frac{\partial T^3}{\partial F} + y - X^1\frac{\partial E^1}{\partial F} - X^1\frac{\partial T^1}{\partial F} - F\frac{\partial E^3}{\partial F} - E^3 = 0$$

In first-best optimum, where $p_i \equiv m_i$, optimal bus price is:

$$(13)\qquad p_3 = \left(C + X^3\frac{\partial T^3}{\partial F} - y + F\frac{\partial O}{\partial F} + F\frac{\partial E^3}{\partial F} + E^3 + X^1\frac{\partial E^1}{\partial F} + X^1\frac{\partial T^1}{\partial F}\right)\frac{F}{X^3}$$

We observe the following: (i) optimal price equals the direct marginal social costs per passenger due to an additional departure; (ii) if frequency has a negligible influence on speed, passengers' travel time and environment, then optimal price equals the only term left on the right hand side, which is average operating cost CF/X^3; (iii) the first-order condition with respect to bus frequency reveals a fact which the first-order condition with respect to bus price did not, i.e., that optimal price for bus passengers actually covers external costs inflicted on motorists and the environment. The larger the external and operating costs are, the lower optimal frequency is and the higher the occupancy rate is, and, consequently (according to the first-order condition with respect to price), the higher optimal price is.

Obviously the first-best optimum implies that profit may in principle be positive or negative. The term $y > 0$ (discussed in detail in paper 1) represents the direct positive effects on passengers in terms of a lower frequency delay, while all other terms on the right hand side in (13) represent costs borne by the bus operator and the travellers, due to a marginal frequency change.

2.4 Alternative policies

Outline of the analysis and definitions of alternative policies

We analyse the implications of superimposing road pricing on each of three alternative policies:

a) "second-best policy",
b) "bus optimum policy",
c) "rule of thumb policy".

The "second-best policy" assumes there is a welfare maximising authority in charge of both public and private transport, subject to the (political) constraint that cars must not be priced. The "bus optimum policy" assumes there is a separate public transport authority, dealing only with the welfare of bus passengers, although bus passengers and operator are affected by cars. The "rule of thumb policy" conditions are not derived from an optimisation process but are characterised by the facts that fares revenues are equal to a fixed proportion of cost and that frequency is set so as to keep occupancy rate fixed. This policy is widely applied both by publicly and privately owned bus companies. In countries like Holland, France and Sweden, local political authorities typically determine fares financing to be e.g. 50% (typical Swedish figure) of costs. In Great Britain, where most local and regional transport has been privatised since 1988, the operators can be assumed to set the maximum profit margin which keeps competitors out of the market. In public companies, the occupancy rate is typically determined by what is considered to be "decent" crowding in various periods. In Stockholm, for instance, the rule for local buses in the peak morning hour is that the ratio of number of passengers to number of seats is 1.5, while in the underground the corresponding figure is 2.0. Private operators can be assumed to set the frequency and occupancy rate that keeps competitors out of the market.

We here employ comparative statics, where car travel price is regarded as an exogenous parameter. By differentiating simultaneously the price and the frequency condition with respect to the policy variables p_3 and F and the parameter p_1, we achieve in principal the following equation system in matrix form (where k_{ij} are partial derivatives):

$$(14) \qquad \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \begin{pmatrix} dp_3 \\ dF \end{pmatrix} = \begin{pmatrix} -k_{13}dp_1 \\ -k_{23}dp_1 \end{pmatrix}$$

Application of Cramer's rule and division by dp_1 yields the following static derivatives, which will be discussed for each alternative policy in the text below:

$$\frac{dp_3}{dp_1} = \frac{k_{23}k_{12} - k_{13}k_{22}}{k_{11}k_{22} - k_{12}k_{21}} \; ; \qquad \frac{dF}{dp_1} = \frac{k_{21}k_{13} - k_{11}k_{23}}{k_{11}k_{22} - k_{12}k_{21}}$$

We assume demand functions to be approximately linear, so that second-order derivatives with respect to demand are approximated to zero. All costs dependent on demand (environmental, congestion and crowding costs) are assumed to be convex in demand, based on the well-known fact that run time, and subsequently exhaust gases, per kilometre are convex in number of vehicles on the road.

We use super- and subscript for first and second-order dérivatives, so that for example:

$$\frac{\partial X^1}{\partial F} \equiv X^1_F \; ; \frac{\partial T^\tau}{\partial F} \equiv T^\tau_F \; ; \frac{\partial T^1_1}{\partial X^1} \equiv T^1_{11} \; ; \frac{\partial O_F}{\partial X^1} \equiv O_{F1} \; ; \frac{\partial O_F}{\partial F} \equiv O_{FF} \; ; \frac{\partial}{\partial X^1}(E^1_1) \equiv E^1_{11}$$

We assume that the direct (substitute) effects on bus and car demand of a marginal change in road pricing are large enough, so that total own-price elasticity, $dX^1/dp_1<0$ and total cross-price elasticity, $dX^3/dp_1>0$, irrespective of the direction of indirect effects via marginal changes in optimal bus price, dp_3/dp_1 and bus frequency, dF/dp_1 (see expressions below, where δ denotes direct effects).

$$\underset{(+)}{\frac{dX^3}{dp_1}} = \underset{(-)}{\frac{\delta X^3}{\delta p_3}}\underset{(+)}{\frac{dp_3}{dp_1}} + \underset{(+)}{\frac{\delta X^3}{\delta p_1}} + \frac{\delta X^3}{\delta F}\frac{dF}{dp_1} \quad ; \quad \underset{(-)}{\frac{dX^1}{dp_1}} = \underset{(+)}{\frac{\delta X^1}{\delta p_3}}\frac{dp_3}{dp_1} + \underset{(-)}{\frac{\delta X^1}{\delta p_1}} + \underset{(-)}{\frac{\delta X^1}{\delta F}}\frac{dF}{dp_1}$$

X^1_1 and X^3_1 will refer throughout to the total effect on demand, i.e., dX^1/dp_1 and dX^3/dp_1, so that effects on demand are only considered when we differentiate with respect to the car price, p_1.

"Second-best policy"

By combining (1) for cost and (9) for marginal cost, $m_3(X^3/F)$ - C in (12) can be written as $X^3(X^3/F)T^3_3$ - $O[X^1, F]$, which is the passengers' marginal cost minus the passenger volume independent operator cost per passenger. Observing that $\partial C/\partial X^3 \equiv B$, the first-order conditions for the "second-best policy" with respect to price and frequency (see (9) and (12)) are thus:

$$\text{(15a)} \quad \frac{\partial w}{\partial p_3} = X^3_3(p_3 - X^3T^3_3 - B) + X^1_3(p_1 - X^1T^1_1 - X^3T^3_1 - FO_1 - X^1E^1_1 - FE^3_1) = 0$$

$$\text{(15b)} \quad \frac{\partial w}{\partial F} = X^3\frac{X^3}{F}T^3_3 - O[X^1,F] - FO_F + y - X^3T^3_F - X^1T^1_F - FE^3_F - E^3 - X^1E^1_F = 0$$

Our assumption that total car and bus demand is constant implies that the first-order conditions in (15) are the same for first-best and second-best optimum, meaning that the analysis of bus price and frequency variations that follows hold equally well for car price variations from any original price, e.g., both no car pricing and optimal car pricing may be regarded as the original situation from which we change car pricing.

The differentials of the first-order conditions with respect to p_3, F and p_1 are:

$$\text{(16a)} \quad \underset{(-)}{X^3_3}\, dp_3 + (-\underset{(-)}{X^3_3}\, X^3\underset{(-)}{T^3_{3F}} +$$

$$+ X^1_3(-O_1 - E^3_1 - X^1T^1_{1F} - X^3T^3_{1F} - FO_{1F} - X^1E^1_{1F} - FE^3_{1F}))dF +$$

$$+ (-X^3_1\, \underset{(-)}{X^3_3}(T^3_3 + X^3T^3_{33}) + X^1_3 - X^3_1 X^1_3 T^3_1 -$$

$$- \underset{(-)}{X^1_1}\, X^1_3(T^1_1 + E^1_1 + T^1_{11}X^1 + T^3_{11}X^3 + FO_{11} + E^1_{11}X^1 + E^3_{11}F))dp_1 \equiv$$

$$\equiv k_{11}dp_3 + k_{12}dF + k_{13}dp_1 = 0$$

(16b) $0dp_3 +$

$$+\Big(-\frac{X^3X^3}{F^2}T^3_3 \overset{(-)}{+ X^3\frac{X^3}{F}T^3_{3F}} - 2O_F - 2E^3_F - FO_{FF} \overset{(-)}{+y_F} - X^3T^3_{FF} -$$

$$- X^1T^1_{FF} - FE^3_{FF} - X^1E^1_{FF}\Big)dF +$$

$$+\Big(X^3_1\big(\frac{2X^3}{F}T^3_3 - T^3_F\big) + X^3_1\, y_3 +$$

$$+\overset{(-)}{X^1_1}(-O_1 - E^3_1 - T^1_F - E^1_F - X^1E^1_{F1} - X^3T^3_{F1} - X^1T^1_{F1} - FO_{F1} - FE^3_{F1}\Big)\, dp_1 \equiv$$

$$\equiv k_{21}dp_3 + k_{22}dF + k_{23}dp_1 = 0$$

Let us examine the signs of the six k_{ij}. Optimum implies that $k_{11}<0$, $k_{22}<0$ and $k_{11}k_{22} - k_{12}k_{21}>0$. The fact that costs are convex in vehicle flow implies that $k_{12}<0$. In k_{13} there is one negative term, $-X^3_1 X^1_3 T^3_1$. Knowing that $X^3_1 \leq |X^1_1|$ and assuming that $T^1_1 \approx T^3_1$ (cars affect the speed of buses and cars approximately equally), we conclude that $k_{13}>0$. Obviously $k_{21}=0$. In k_{23} there is only one negative term, $-X^3_1 T^3_F$. Compare this term with $X^1_1 T^1_F$ which is positive. Since $X^3_1 \leq |X^1_1|$ and T^1_F and T^3_F are approximately equal in size (a marginal bus affects the speed of buses and cars approximately equally), we conclude that $k_{23}>0$.

The comparative static derivatives are thus (assuming the second-order condition to be met):

$$\frac{dp_3}{dp_1} = \frac{\overset{(-)}{k_{23}k_{12}} - \overset{(-)}{k_{13}k_{22}}}{\underset{(-)\,(-)}{k_{11}k_{22}} - \underset{(-)\,(0)}{k_{12}k_{21}}} \quad ; \quad \frac{dF}{dp_1} = \frac{\overset{(0)}{k_{21}k_{13}} - \overset{(-)}{k_{11}k_{23}}}{\underset{(-)\,(-)}{k_{11}k_{22}} - \underset{(-)\,(0)}{k_{12}k_{21}}}$$

It follows that $dF/dp_1>0$, while dp_3/dp_1 seems to be able to take on both signs. $dp_3/dp_1<0$ can, however, be ruled out as a feasible solution, since this would mean that insufficient road pricing is met by a bus price exceeding the first-best optimal and a bus frequency below the first-best optimal, which is obviously absurd. The magnitudes of the various partial derivatives must thus be such that $dF/dp_1>0$ and $dp_3/dp_1>0$. Introduction of road pricing in this situation would increase both bus frequency and bus price. On the other hand, if we move from a first-best situation with optimal road pricing to a second-best situation without it, suboptimal pricing of cars should be met by a decrease in bus price but also a decrease in bus frequency. The intuitive reason why frequency should not be used in order to reduce excessive car traffic is that increased frequency has a negative impact on congestion and the environment, at the same time as increased frequency decreases the positive external effect. Since the introduction of road-pricing would increase both bus frequency and bus price, we are not able to tell whether bus passengers would be better or worse off.

"Bus optimum policy"

The objective function now includes only costs and surplus related to the bus mode, implying that the first-order conditions (10) become:

$$(17)\qquad \begin{pmatrix} X_3^3 & X_3^4 \\ X_4^3 & X_4^4 \end{pmatrix} \begin{pmatrix} d_3 \\ d_4 \end{pmatrix} = 0$$

This scheme has the optimal solution $d_3 = d_4 = 0$, i.e., optimal bus prices equal their respective marginal costs.

The first-order conditions are (observing that all costs affecting cars are missing):

$$(18a)\qquad \frac{\partial w}{\partial p_3} = X_3^3(p_3 - X^3T_3^3 - B) = 0$$

$$(18b)\qquad \frac{\partial w}{\partial F} = X^3\frac{X^3}{F}T_3^3 - O[X^1, F] + y - FE_F^3 - E^3 - FO_F - X^3T_F^3 = 0$$

Differentiating conditions (18) with respect to p_3, F and p_1 yields:

$$(19a)\qquad \overset{(-)}{X_3^3}\, dp_3 + (-\overset{(-)}{X_3^3}\ X^3\overset{(-)}{T_{3F}^3})dF + (-X_1^3\ \overset{(-)}{X_3^3}(T_3^3 + X^3T_{33}^3))dp_1 \equiv k_{11}dp_3 + k_{12}dF + k_{13}dp_1 = 0$$

(19b) $0dp_3 +$

$$+\left(-\frac{X^3X^3}{F^2}T_3^3 + X^3\frac{X^3}{F}\overset{(-)}{T_{3F}^3} - 2O_F - 2E_F^3 - FO_{FF} + \overset{(-)}{y_F} - X^3T_{FF}^3 - FE_{FF}^3\right)dF +$$

$$+\left(X_1^3\left(\frac{2X^3}{F}\overset{(-)}{T_3^3} - T_F^3\right) + \overset{(-)}{y_1} + X_1^1(-O_1 - E_1^3 - X^3T_{F1}^3 - FO_{F1} - FE_{F1}^3)\right)dp_1 \equiv$$

$$\equiv k_{21}dp_3 + k_{22}dF + k_{23}dp_1 = 0$$

The comparative static derivatives are thus (assuming the second-order condition to be met):

$$\frac{dp_3}{dp_1} = \frac{\overset{(-)}{k_{23}}\overset{(-)}{k_{12}} - k_{13}k_{22}}{\underset{(-)}{k_{11}}\underset{(-)}{k_{22}} - \underset{(-)}{k_{12}}\underset{(0)}{k_{21}}}\ ; \qquad \frac{dF}{dp_1} = \frac{\overset{(0)}{k_{21}}\overset{(-)}{k_{13}} - k_{11}k_{23}}{\underset{(-)}{k_{11}}\underset{(-)}{k_{22}} - \underset{(-)}{k_{12}}\underset{(0)}{k_{21}}}$$

Compared to the "second-best policy", the only important difference in partial derivatives is that the effects on cars of a marginal change in frequency are missing in k_{22}. The implication of this fact is that for the "bus optimum policy" it is likely that k_{22} has a smaller absolute value than for the second-best policy. Comparing these two policies then indicates that the bus-optimum policy means a larger increase in frequency and a smaller increase in price when road-pricing is introduced. The intuitive reason is that under the bus-optimum policy an increase in frequency is assumed to cause no harm on car users and the environment.

"Rule of thumb policy"

A common policy applied by politically controlled bus companies is that financial grants are determined so as to cover a fixed percentage of costs ($\omega < 1$), or make a "decent" profit ($\omega \geq 1$). Private operators can be assumed to determine a reasonable profit level ($\omega \geq 1$), which corresponds to the "normal" profit they can make without losing ground to other actual or potential operators. Both types of operators can also be assumed to determine a "decent" occupancy rate for peak and off-peak periods, based either on convenience or on competitive commercial criteria. The price and frequency conditions are then:

(20a) $\quad p_3X^3 - \omega(BX^3 + FO[X^1, F]) = 0$

(20b) $\quad F - \beta X^3 = 0$

where ω is the cost coverage requirement and β is the inverse of the fixed load factor, i.e., $\beta = 1/q \equiv F/X^3$, for fixed F and X^3.

Differentiating conditions (20) with respect to p_3, F and p_1 yields:

(21a) $\quad X^3\, dp_3 + (-\omega O - \omega FO_F)dF + (p_3X_1^3 - \omega BX_1^3 \overset{(-)}{-\omega FO_1X_1^1})dp_1 \equiv k_{11}dp_3 + k_{12}dF + k_{13}dp_1 = 0$

(21b) $\quad 0dp_3 + dF - \beta X_1^3 \equiv k_{21}dp_3 + k_{22}dF + k_{23}dp_1 = 0$

In (21) $k_{11}>0$, $k_{12}<0$, $k_{13}>0$ (since $p_3X_1^3 - \omega BX_1^3 > 0$ according to (20a)), $k_{21}=0$, $k_{22}>0$ and $k_{23}<0$. Then:

$$\frac{dp_3}{dp_1} = \frac{\overset{(-)}{k_{23}k_{12}} - \overset{(-)}{k_{13}k_{22}}}{k_{11}k_{22} - \underset{(0)}{k_{12}k_{21}}} \; ; \; \frac{dF}{dp_1} = \frac{\overset{(0)}{k_{21}k_{13}} - \overset{(-)}{k_{11}k_{23}}}{k_{11}k_{22} - \underset{(0)}{k_{12}k_{21}}} > 0.$$

Obviously $dF/dp_1>0$, while the sign of dp_3/dp_1 is less clear. Therefore we substitute (20b) into (20a) and divide by X^3, to obtain:

(20a') $\quad p_3 - \omega(B + \beta O[X^1, \beta X^3]) = 0$

Differentiating (20a') with respect to p_3 and p_1 yields:

(22) $\quad X^3dp_3 + \omega\beta X^3(\overset{(-)}{-O_1X_1^1} - \beta O_FX_1^3)dp_1 = 0$

which means that $dp_3/dp_1 = \omega\beta(O_1X_1^1 + \beta O_FX_1^3)$.

Since $X_1^3 < |X_1^1|$, and O_F is approximately twice the size of O_1 (due to the fact that a bus is bigger than a car) and β is the inverse of the number of seats in a bus, it follows immediately that $dp_3/dp_1<0$, given that O_F and O_1 are positive, i.e., given that there is congestion. Where there is no congestion, $dp_3/dp_1=0$.

The effects of the "rule of thumb policy" when car pricing is introduced is thus found to depend on the level of congestion. If congestion is negligible, bus price is unchanged while frequency is increased. Where there is congestion, frequency is still increased but bus price is decreased. Introduction of road pricing would in either case benefit the bus passengers. The intuitive explanation is that the excessive demand for car travel corresponds to low bus travel demand, which in turn motivates a low frequency. Since only cost coverage, and no consumer benefit, is taken into account, price is high in order to compensate for the loss of passengers. We can also turn this result around and say that where cars are not priced and where the "rule of thumb policy" is adopted, bus passengers are definitely losers; where there is congestion, they are worse off both in terms of a higher price and a lower frequency than they would be with car pricing.

2.5 Conclusions

We have discussed the implications in terms of price and frequency for bus passengers, with and without road pricing, for three different transport policies. Earlier studies of road pricing have focused on corrective second-best pricing in the absence of optimal road pricing, and correctly so, but have dealt with neither the consequences of alternative policies nor the consequences for bus passengers in terms of service frequency. The analysis in this paper was made possible by use of a model including joint optimisation of price and frequency of public transport and taking into account interrelations between modes in terms of demand and costs, determined by congestion and negative environmental effects.

A first-best optimum was derived to serve as a point of reference. Optimal price covers marginal social costs, including environmental costs, caused by an additional traveller. Marginal environmental costs caused by additional car users arise directly from the increased number of cars. Increases in marginal social costs caused by additional bus users arise indirectly through an increased optimal service frequency, and hence increased number of buses.

We first examined a "second-best policy", assuming a welfare maximising authority to be in charge of both roads and public transport and to aim at optimal public transport pricing, given that cars cannot be priced. For this policy, introduction of road pricing would increase both bus frequency and bus price, meaning that we are not able to tell whether bus passengers would be better or worse off. On the other hand, if we move from a first-best situation with optimal road pricing to a second-best situation without it, suboptimal pricing of cars should be met by a decrease in bus price but also a decrease in bus frequency.

The second policy discussed was the "bus optimum policy". This policy assumes that the public transport authority optimises bus price and frequency and that the flow of cars affects both the operator and the passengers, but that the welfare of motorists is disregarded. Where this policy is adopted, we obtain in principle the same outcome as for the "second-best policy". But, since the authority is not concerned with the fact that buses affect the environment and congestion, the bus-optimum policy means a larger increase in frequency and a smaller increase in price when road-pricing is introduced.

The third policy discussed was the "rule of thumb policy". This policy is not based on optimisation, but is characterised by the fact that price is set so that fares revenues cover a fixed percentage of costs (> or < 100%) and that frequency is set to keep a fixed load factor. This policy is believed to be the most common irrespective of whether the buses are publicly or privately operated. The implications of the rule of thumb policy were shown to differ with respect to the level of road congestion. Where congestion is negligible, introduction of road pricing (then motivated by environmental costs only) would not change the bus price but would increase frequency. Where there is congestion, introduction of road pricing would, in fact, benefit the bus passengers both through a lower price and a higher frequency.

3 COMPUTER SIMULATIONS FOR STOCKHOLM

3.1 Introduction

We have used computer simulation techniques to describe the outcome if pricing principles were moved towards first-best pricing, both for private and public transport.

The computer simulations and analysis of car travel pricing (using the Emme 2 system) have been carried out by the consultancy firm Transek, and the computer simulations of public transport (using the Vips system) by the Stockholm County owned public transport company SL, both on behalf of the Swedish Institute for Transport and Communications Analysis. The link between these two analyses and the principles for public transport prices and frequencies have been the responsibility of the author of this paper.

The analysis comprises of the following steps: 1) Car assignment "software based" calculation of time savings, revenues and loss of demand for private transport if a differentiated zone-based road pricing scheme is introduced in Stockholm's inner city. 2) An estimation of appropriate fare increases on public transport, given 1). Since step 1) provides the number of former car drivers who convert to public transport for each O-D pair, and step 2) provides a change in public transport demand for each O-D pair, the public transport system is in step 3) re-designed, in terms of frequencies, in order to meet the overall increase in public transport demand. 4) The public transport assignment software Vips is used for calculation of the consequences concerning public transport, in terms of consumers´ and producers´ surplus, exhaust pollution and total social welfare.

The results of the project have been used by the Swedish Parliamentary Transport and Communication Commission for their work on a new Swedish transport policy.

3.2 Assumptions and prerequisites for the Stockholm case study

Measures

The following combination of measures is assumed:
- road user charges in the Stockholm area
- increased public transport fares to and from the inner city of Stockholm
- increased public transport frequency in proportion to the increased public transport demand.

The road user charges are meant to reflect the negative external effects on each link in the Stockholm area, not only in the inner city. The negative external effects are calculated based on the recommendations made by SIKA in collaboration with Vägverket (the Swedish National Road Authority) and Banverket (the Swedish Rail Track Authority). These external effects comprise of exhaust gases, climate gases, noise, congestion and accidents.

With respect to public transport service frequency we have not necessarily aimed to calculate the optimal frequencies. Instead we have followed the principle of "rule-of thumb", which was discussed in section 2. The reason is that we have taken for granted the principle applied by SL. The consequence is that we have changed frequency in proportion to the calculated change in demand due to the road-user charges.

With respect to public transport fares we had the ambition, jointly with SL, to simulate a pricing structure which is more related to the marginal costs than the present structure. The proposed structure assumes a zonal based travel card instead of the present uniform area-wide pricing.

It is assumed that revenues from road pricing and public transport fares are used for tax decreases, which give rise to an excess burden gain.

Values of time

The time values shown in table 1, below, are recommended by the Swedish Institute for Transport and Communications Analysis, SIKA, for national infrastructure planning.
1 SEK is approximately 0,09 Pounds Sterling (GBP).

	Car users Values SEK/h	Proportions %	Public transport users Values SEK/h	Proportions %
Lorries	270	8		
Business	140	19	140	3
Home-work	35	35	35	47
Leisure, school	26	38	26	50
Mean	73		32	

Table 1: Values of time

Road Pricing

Based on the marginal external costs the average peak price in the Stockholm area would be SEK 1.40/veh.km., 80% of which is based on congestion costs, 15% on accident costs and 5% on environmental costs. The maximum price in the inner city during the morning peak would be SEK 6.10/veh.km.

Public transport pricing

In accordance with the discussion in section 2 the following principles are used:

1. The price is related to the in-vehicle crowding and time spent on each link
2. The price increases with the boarding time, meaning that cash tickets are priced higher than pre-purchased tickets which in turn are priced higher than travel cards.

For these purposes a zonal system is assumed to be appropriate, where the zones are smaller closest to the centre and larger further from the centre. Such zones are already implemented for cash and pre-purchased tickets but not for travel cards. The figure below illustrates schematically the proposed travel card zoning system for travel cards.

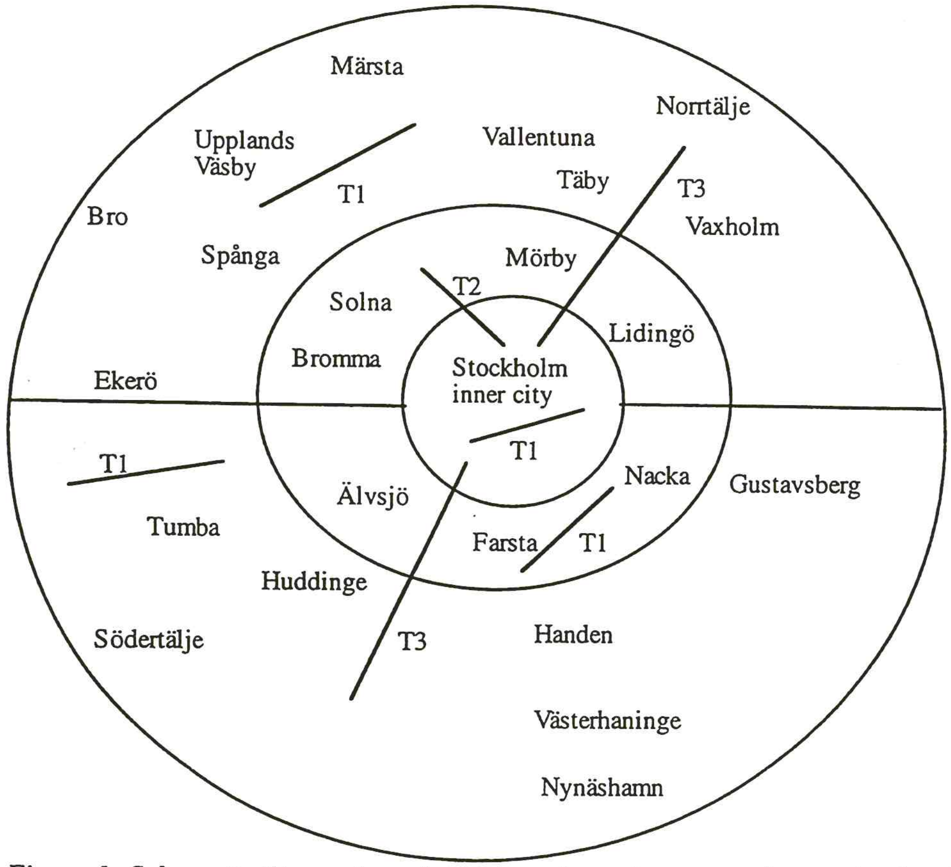

Figure 1. Schematic illustration of travel card zonal system, where price levels are marked T1, T2 and T3.

According to Jansson (1991) the optimal deficit for public transport in central to inner areas in Stockholm is around 50% of operating costs. Since the tax financed proportion today is around 60%, the price level should be increased by some 25%.

The proposed prices are as follows:.

Travel in 1 zone,	T1:SEK 355/month (equal to the area-wide uniform price in 1996)
Travel in 2 zones,	T2:SEK 425/month
Travel in 3+ zones,	T3:SEK 495/month

These prices mean that the average price of a travel card is increased by SEK 1.90 per journey which is calculated to give the extra revenue of SEK 330 million per year.

Demand and service frequency

Road pricing is calculated to increase public transport demand by 4%, which means that we assume 4% increase in service frequency. This corresponds to an extra 18 630 vehicle kilometres between 6 and 9 PM on a yearly basis, distributed as follows:

Commuter train	Underground	Other train	Diesel bus	Ethanol bus
5 480	5 990	1 310	7 170	250

Table 2: Increase of supply, 1000 vehicle kilometres

Speed in the inner city of Stockholm

The average speed in the inner city is calculated to increase by 20%. Since 20% of the round trip time is stop time, the net increase is 13% and the number of buses is reduced by 13%.

External costs

Here we present the recommendations for external costs in terms of exhaust gases, climate gases, noise, maintenance and accidents, as calculated by SIKA for bus and by SL for commuter train and underground.

	Bus Ethanol	**Bus Diesel, Euro 1**	**Commuter train**	**Underground Tram**
Production, distribution	4,50	1,25	1,25	0,71
Exhaust gases, except CO_2	0,80	0,80	0	0
Accidents	0,72	0,72	1,20	1,20
Noise	0,29	0,29	2,55	2,55
Maintenance	0,27	0,27	1,85	1,85
Sum	1,28	2,09	5,60	5,60
CO_2 cost (derived)	0	0,48	0	0
Total external costs	2,08	2,57	5,60	5,60
Tax, energy	0	0,79	0	0
Tax CO_2	0	0,47	0	0
Track charges			1,85	1,85
Total tax excl. VAT	0	1,25	1,85	1,85
External costs - tax	2,08	1,34	3,75	3,75

Table 3: External costs and taxes per vehicle (train) kilometre

3.3 Results

Social welfare

The figures below show toal benefits, costs and benefits minus costs, distributed by winners and losers.

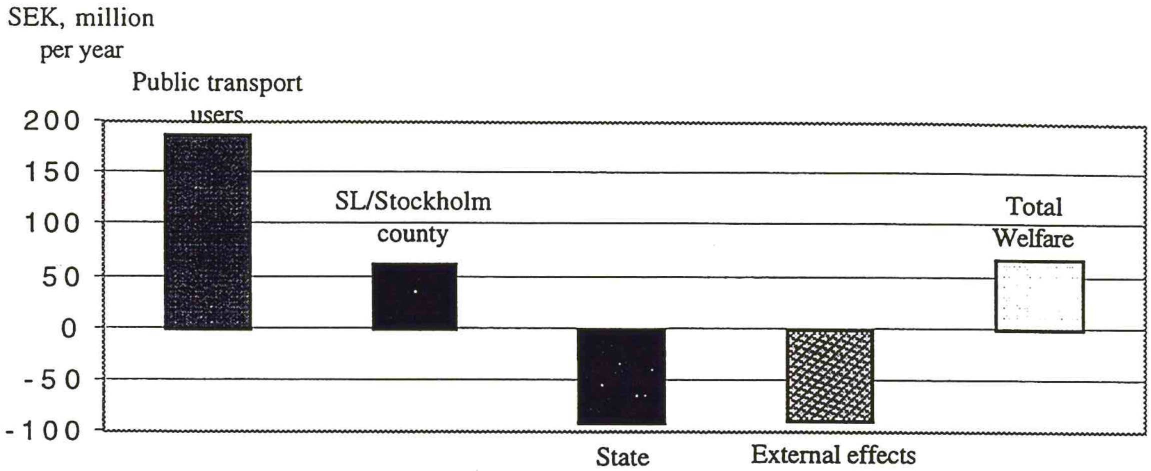

Figure 2: Distribution of welfare with respect to public transport changes

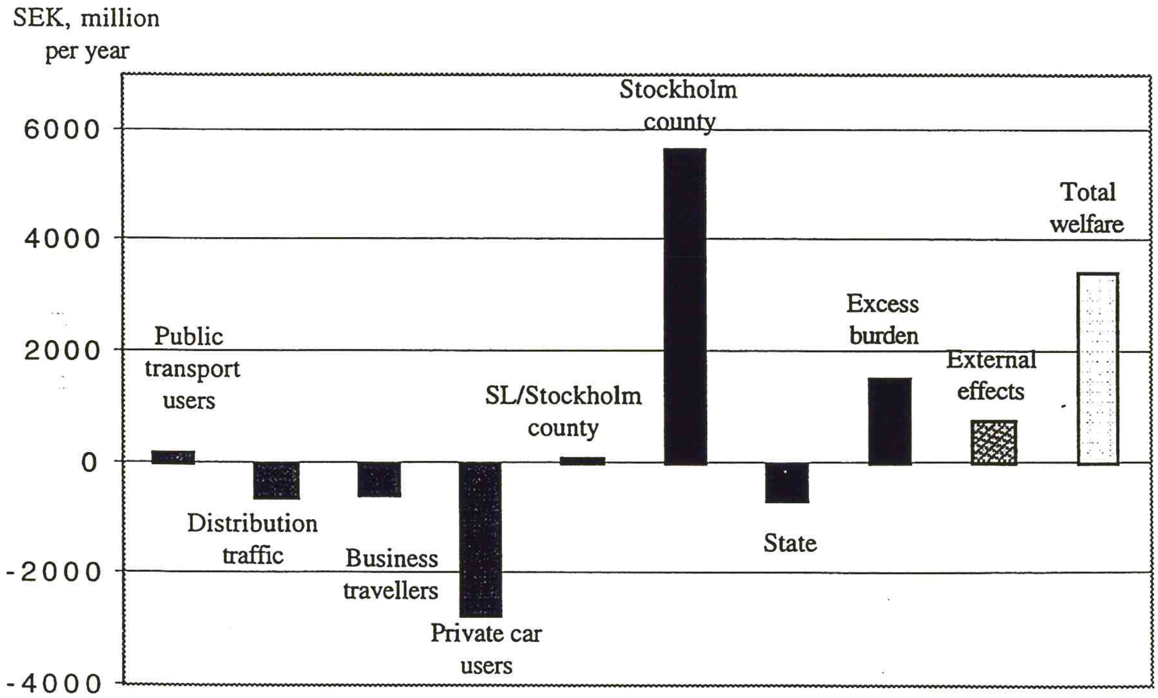

Figure 3: Distribution of welfare with respect to road pricing and public transport price and frequency changes

Most notable is that the majority of the effects stem from road pricing. Of the total welfare effect, approximately SEK 3 500 million, the effect of public transport pricing and frequency change is only around SEK 70 million, which equals to only 2 per cent.

Public transport users would gain slightly, while motorists would lose substantially. The main winners are the tax payers , via the authority that collects the road-user charges.

Note that the net benefit would be positive even if exhaust gases and accidents were not valued at all. While in many cases measures to improve the environment cause social net costs, the environment in this case can be improved not only without cost but also yielding a net social gain.

Financial result

SL would gain from the increased public transport fares and the county or the city of Stockholm would gain from the road user charges. The state would lose from loss of taxes. See the table below

	Finances SEK, million/year
SL	60
Stockholm county	5 650
State	-710
Net	+4 430

Table 4: Financial outcome for SL, Stockholm county and the state

Distributional issues

It would be possible to compensate the car users by decreasing the yearly vehicle tax. It may seem unfair, though, to give all the revenues to the car owners since the revenues would amount to some SEK 10 000 per year while the annual tax today is below SEK 1 000.

A large amount of the revenues could be used for income tax reductions. If all of the revenues, amounting to around SEK 5 000 million per year, would go to tax reductions, each income owner in the county would get a reduction of around SEK 6 000 per year.

3.4 Conclusions

The combination of road pricing and increased public transport fares and frequency would greatly benefit society as a whole.

Most of the benefits stem from road pricing, the revenues of which could be used for efficiency improving tax reductions.

The move from second-best to first-best pricing and increased service frequency of public transport, which is economically justified by introduction of road pricing, is beneficial both for the public transport users and the operator.

The measures imply that a "tax shift" would be socially beneficial, even if environment effects are not valued at all.

References

Glaister, S. (1974): "Generalised Consumer Surplus and Public Transport Pricing". *The Economic Journal*, December.

Greater London Council (1973): "Transport in London"

Jansson, K. (1991): "Efficient Prices and Quality in Public Transport", Department of Economics, Stockholm University, 1991.

Jansson, J. O. (1979): "Marginal Cost Pricing of Scheduled Transport Services - A Development and Generalisation of Turvey-Mohrings Theory of Optimal Bus fares". *Journal of Transport Economics and Policy*, vol. 13.

Jansson, J. O. (1980): *Transport System Optimisation and Pricing*. EFI, Stockholm School of Economics, Stockholm.

Mohring, H. (1972): "Optimisation and Scale Economics in Urban Bus Transportation". *The American Economic Review*, September.

Panzar, J. C. (1979): "Equilibrium and Welfare in Unregulated Airline Markets". *The American Economic Review*, May.

Storstadstrafikkommittén (1989): "Storstadstrafik 5". SOU 1990:16, Stockholm

Turvey, R. and Mohring, H. (1975): "Optimal Bus Fares". *Journal of Transport Economics and Policy*, vol. IX.

Walters, A. (1968): "Road User Charges". *International Bank for Reconstruction and Development*

APPENDIX

For the objective function, $w[\Sigma_i S^i[G^i[\mathbf{p},F]] + \Sigma_i \pi^i - \Sigma_i E^i]$, knowing that $\partial S^i/\partial G^i = -X^i$ (see (7)), first-order conditions with respect to a price p_j and frequency F can, letting m denote social costs due to a marginal demand change, be calculated according to :

$$\frac{\partial w}{\partial p_i} = \Sigma_j \frac{\S w}{\S X^j}\frac{\partial X^j}{\partial p_i} \equiv \Sigma_j\left(\frac{\S S^j}{\S G^j}\frac{\S G^j}{\S X^j} + \frac{\S \pi^j}{\S X^j} - \frac{\S E^j}{\S X^j}\right)\frac{\partial X^j}{\partial p_i} \equiv \Sigma_j(p_j - m_j)\frac{\partial X^j}{\partial p_i} = 0$$

$$\frac{\partial w}{\partial F} = \Sigma_j \frac{\S w}{\S X^j}\frac{\partial X^j}{\partial F} + \frac{\delta w}{\delta F} \equiv \Sigma_j(p_j - m_j)\frac{\partial X^j}{\partial F} + \frac{\delta w}{\delta F} = 0$$

where, in order to distinguish between various types of derivatives, § is used for the marginal effects with respect to demand and generalised cost, δ is used for the direct marginal effect on welfare with respect to frequency, and ∂ is used for partial derivatives. Once the first-order condition with respect to demand or price is derived, we can directly insert $\Sigma_j(p_j - m_j)$ from this derivation when deriving with respect to frequency, so that we only have to calculate the direct effect of frequency, $\delta w/\delta F$.

We here present the derivation of the first-order conditions for the objective function (8) with respect to p3 and F. The corresponding derivation with respect to car price is analogous.

First-order condition with respect to p3:

(A1) $$\frac{\partial w}{\partial p_3} = \frac{\partial S^3}{\partial G^3}\frac{\partial G^3}{\partial p_3} + \frac{\partial S^1}{\partial G^1}\frac{\partial G^1}{\partial p_3} + p_1\frac{\partial X^1}{\partial p_3} + p_3\frac{\partial X^3}{\partial p_3} +$$

$$+ F\left(X^3 - \frac{\partial C^3}{\partial X^3}\frac{\partial X^3}{\partial p_3} - \frac{\partial C^3}{\partial X^1}\frac{\partial X^1}{\partial p_3} - \frac{\partial E^3}{\partial X^1}\frac{\partial X^1}{\partial p_3}\right) - X^1\frac{\partial E^1}{\partial X^1}\frac{\partial X^1}{\partial p_3} - E^1\frac{\partial X^1}{\partial p_3} = 0$$

where

$$\frac{\partial G^1}{\partial p_3} = \frac{\partial T^1}{\partial X^1}\frac{\partial X^1}{\partial p_3}$$

$$\frac{\partial G^3}{\partial p_3} = 1 + \frac{\partial T^3}{\partial X^3}\frac{\partial X^3}{\partial p_3} + \frac{\partial T^3}{\partial X^1}\frac{\partial X^1}{\partial p_3}$$

Using (7) and writing derivatives by using super- and subscript, so that for example

$\frac{\partial C}{\partial X^3} \equiv C_3$, $\frac{\partial X^1}{\partial p_3} \equiv X^1_3$ and $\frac{\partial T^3}{\partial X^3}\frac{\partial X^3}{\partial p_3} \equiv T^3_3 X^3_3$, the first-order condition can be expressed as:

(A2) $$\frac{\partial w}{\partial p_3} = X^1_3 (p_1 - X^1 T^1_1 - X^3 T^3_1 - FC_1 - X^1 E^1_1 - E^1 - FE^3_1) +$$
$$+ X^3_3(p_3 - X^3 T^3_3 - FC_3) \equiv \Sigma_j X^j_3(p_j - m_j) = 0$$

where $p_j - m_j = 0$ represents first-best.

Analogous derivations of first-order condition with respect to car price yield equations (11) in section 2.3.

The first-order condition with respect to frequency, F, is as follows:

$$\text{(A3)} \quad \frac{\partial w}{\partial F} = \frac{\delta S3}{\delta F} + \frac{\delta S1}{\delta F} - C - F\frac{\delta C}{\delta F} - E^3 - F\frac{\delta E3}{\delta F} - X1\frac{\delta E1}{\delta F} +$$

$$+ (p_1 - m_1)\frac{\partial X1}{\partial F} + (p_3 - m_3)\frac{\partial X3}{\partial F} = 0$$

where (see (6)):

$$\frac{\delta S1}{\delta F} \equiv \frac{\delta S1}{\delta G1}\frac{\delta G1}{\delta F} \equiv - X^1 \frac{\delta T^1}{\delta F}$$

$$\frac{\delta S3}{\delta F} \equiv \int_0^{1/F} s^3[G^3[p_3,F,t]]dt - F(1/F^2)s^3[G[p_3,F,1/F]] + F\int_0^{1/F}\frac{\partial s^3}{\partial G^3}\frac{\partial G^3}{\partial F}dt$$

and where

$$F\int_0^{1/F}\frac{\partial s^3}{\partial G^3}\frac{\partial G^3}{\partial F}dt \equiv F\int_0^{1/F}\frac{\partial s^3}{\partial G^3}\frac{\partial T^\tau}{\partial F}dt + F\int_0^{1/F}\frac{\partial s^3}{\partial G^3}\left(\frac{\partial \phi^3}{\partial R}\frac{-X^3}{\sigma F^2}\left(b\frac{X^3}{F} + \gamma r^3\right) + \phi^3\left(b\frac{-X^3}{F^2} + \gamma\frac{\delta r^3}{\delta F}\right)\right) \equiv$$

$$\equiv F\int_0^{1/F}\frac{\partial s^3}{\partial G^3}\frac{\partial T^\tau}{\partial F}dt + \frac{X^3}{F}\left(X^3\left(\frac{\partial \phi^3}{\partial R}\frac{1}{\sigma F}\left(b\frac{X^3}{F} + \gamma r^3\right) + \phi^3\frac{b}{F}\right)\right) - X^3\phi^3\gamma\frac{\delta r^3}{\delta F}$$

We use y to denote the change in consumers' surplus due to a marginal change in frequency, i.e.:

$$y \equiv \int_0^{1/F} s^3[G^3[p_3,F,t]]dt - F(1/F^2)s^3[G[p_3,F,1/F]] + F\int_0^{1/F}\frac{\partial s^3}{\partial G^3}\frac{\partial T^\tau}{\partial F}dt$$

We use:

$$\frac{\delta T^3}{\delta F} \equiv \phi^3\gamma\frac{\delta r^3}{\delta F}$$

to denote the road congestion cost borne by bus passengers caused by an additional bus.

According to (4), (7) and (10) we have:

$$F\frac{\delta C}{\delta F} \equiv -B\frac{X^3}{F} + F\frac{\delta O}{\delta F}$$

$$m_3 \equiv X^3\left(\frac{\partial \phi^3}{\partial R}\frac{1}{\sigma F}\left(b\frac{X^3}{F} + \gamma r^3\right) + \phi^3\frac{b}{F}\right) + B$$

(A3) can then be rewritten as:

$$\text{(A4)} \quad \frac{\partial w}{\partial F} = (p_1 - m_1)\frac{\partial X1}{\partial F} + (p_3 - m_3)\frac{\partial X3}{\partial F} -$$

$$- E^3 - C - F\frac{\delta O}{\delta F} - F\frac{\delta E^3}{\delta F} - X^3\frac{\delta T^3}{\delta F} - X^1\frac{\delta E^1}{\delta F} - X^1\frac{\delta T^1}{\delta F} + \frac{X^3}{F}m_3 + y = 0$$

Let us denote $d_i \equiv p_i - m_i$. Outside first-best optimum, where p_1 is no policy variable, (11) yields that $d_3 \equiv -(X_3^1/X_3^3)d_1$. We then have:

$$d_1\frac{\partial X^1}{\partial F} + d_3\frac{\partial X^3}{\partial F} \equiv d_1 X_F^3\left(\frac{X_F^1}{X_F^3} - \frac{X_3^1}{X_3^3}\right)$$

Our assumption that changes in bus price and frequency mainly cause people to switch between the bus and car alternatives, while the effect on total travel demand for these modes is negligible, means that $X_F^1 \approx X_F^3$, $X_3^1 \approx X_3^3$ and that $\left(\frac{X_F^1}{X_F^3} - \frac{X_3^1}{X_3^3}\right) \approx 0$, so that

$$d_1\frac{\partial X^1}{\partial F} + d_3\frac{\partial X^3}{\partial F} \approx 0.$$

The first-order condition with respect to frequency is then as follows both for first-best and non-first-best:

$$\text{(A5)} \quad \frac{\partial w}{\partial F} = -E^3 - C - F\frac{\delta O}{\delta F} - F\frac{\delta E^3}{\delta F} - X^3\frac{\delta T^3}{\delta F} - X^1\frac{\delta E^1}{\delta F} - X^1\frac{\delta T^1}{\delta F} + \frac{X^3}{F}m_3 + y = 0$$

The effects of a marginal increase in frequency are represented as follows: E^3 and $F(\delta E^3/\delta F)$ are direct environmental costs, $X^1(\delta E^1/\delta F)$ are environmental costs via cars, C is the operating cost per departure, $F(\delta O/\delta F)$ is the effect on operating cost through congestion, $X^1(\delta T^1/\delta F)$ is the cost borne by car users, $(X^3/F)m_3$ is the operator's and the passengers' savings in riding time costs through higher speed and y is the reduction in frequency delay cost. Note that $\delta T^3/\delta F$ ($\equiv \phi^3\gamma\delta r^3/\delta F$) represents the increase in riding-time cost due to increased road congestion, while the reduction in crowding cost that follows from a higher frequency is absorbed in $(X^3/F)m_3$.

In optimum $p_i = m_i$; i=1,3, implying that bus passengers pay a price equal to m_3, which can then be replaced by p_3. We can then express the first-order condition with respect to frequency in terms of optimal price (knowing that all derivatives with respect to frequency represent direct effects we now use ∂ instead of δ for these derivatives):

$$\text{(A6)} \quad p_3 = \left(C - y + F\frac{\partial O}{\partial F} + F\frac{\partial E^3}{\partial F} + E^3 + X^1\frac{\partial E^1}{\partial F} + X^3\frac{\partial T^3}{\partial F} + X^1\frac{\partial T^1}{\partial F}\right)\frac{F}{X^3}$$