OPTIMISATION OF PUBLIC TRANSPORT SERVICES AND CENTRAL AREA CAR PRICING

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ABSTRACT

A modelling framework for optimal pricing and financing is presented. The framework is suitable for urban bimodal contexts with central area car pricing schemes and subsidised public transport services. The problem is formulated as system surplus maximisation. The paper discusses how the formulation adopted can be derived from the original bilevel problem which considers supply at the outer level and demand at the inner level. The theoretical underpinnings of the utility maximisation framework used for the formulation of the demand problem are presented. The formulation of the representative traveller's utility maximisation problem, which only produces aggregate demand in case of congested networks and that retrieves frequency, destination, mode and route choices according to nested logit model, is provided. The cases of user homogeneity and heterogeneity and of non-interacting and interacting modes are considered. The paper presents the transformation needed to obtain the formulation of the system surplus maximisation problem from that of the representative traveller's utility maximisation problem. The paper concludes showing how to derive practical formulations of the problem from the theoretical solution.

1. INTRODUCTION

The rapid motorisation that began in the 1950s was followed in the 1960s by a series of studies that theorised the need for an ever greater increase in the supply of infrastructures to meet the growing demand for travel in private cars. According to this reasoning, the role of public transport services was destined to become, over time, relatively modest if not marginal. By the 1970s Hillman et al.[1], Plowden and Schaeffer [2] and Sclar[3], to name a few, were contesting the technical and economic validity of this approach, which was, they maintained, self-fulfilling: the policies adopted were such as to make the predictions come true.Today no one disputes (Goodwin[4]), either the impossibility of increasing the supply of road infrastructures that can satisfy the demand or the necessity of adopting provisions that encourage the use of public transport to reduce rush-hour congestion.

In this paper long-term intervention policies, tending to identify the best endowment of road infrastructures and transport services will not be addressed; instead, we will deal with short-term intervention policies aimed at making the best use of the existing networks in a bimodal context, private car and rapid transit, by introducing some scheme of *pricing and financing* so as to reduce the economic disbenefits due to the discrepancies between the average private costs and the social marginal costs. In fact, when traffic flow builds up and congestion occurs, the Road Space (RS) becomes a scarce good used in competition and, as such, an economic good whose use wants a price to be paid.

In order to identify the way to be followed to make the best use of the existent RS in congested urban areas, some points are to be clarified. It must first of all be stated whether or not is RS a common good, then what functions is the RS conceived to fulfil and finally who must pay for RS provision and maintenance. With reference to actual situations it seems logical to assume that road network and, consequently, RS is an integral part of any urban area; a component, among many others, characterising the structure and the quality of the urban area itself. It follows then that:

a) RS is a common good, whose best use is one of the Local Government's competence;
b) all of the functions of RS, other then those connected with traffic demand, are paid for by means of general taxation;
c) the payment connected with the last use must be determined in the context of traffic equilibrium optimisation problems, whose solution must then be based on some socio-economic criterion, say, just to clear up the idea, welfare maximisation.

In modelling urban travel demand, the *behavioural* approach, or equivalently the *trip consumer approach*, will be adopted where an individual traveller may be considered a consumer of *trips* just as he is a consumer of other *goods*. In this way travel demand can be modelled within the well established microeconomic theory and methods, where pricing and financing schemes, as well as other issues relevant to urban transport policy may be addressed rigorously and where it is possible to address in a totally consistent manner the integration of the supply and demand sides of travel and to identify the resulting equilibrium.

Generally speaking, we can say that the supply side of the problem consists of some social objective function to be maximized subject to, among other constraints, the users behaviour, who aim at optimise their own objective functions, representing the demand side of the problem. This means that determining optimal supply requires solving a constraint optimisation problem in which some of the constraints take the form of another optimisation problem. In fact, as it will be seen in the next section, the demand side of the problem, when, as in the present case, there is congestion, can only be formulated as the solution of the *Representative Traveller's Utility Maximisation Problem*(R.T.'s U.M.P.).

Let the vector **Q**, representing the traffic flow pattern, be split up into two components, $\mathbf{Q}_1$ and $\mathbf{Q}_2$, to remember that the problem we will deal with refers to a bimodal context: private car and transit, respectively. In a similar fashion let the vector Y, representing the resources consumed, be split up into two components, $\mathbf{Y}_s$ and $\mathbf{Y}_u$, to signify that the first ones, which are variables of the socio-economic type, though relevant with respect to the supply side, are not included in the demand side of the problem, where the ones variables affecting the traveller's choices only are included. The problem can then be formally represented as:

$$\begin{aligned} &\mathrm{Max} f(\mathbf{Q}_1,\mathbf{Q}_2;\mathbf{Y}_s,\mathbf{Y}_u) && a) \\ &\text{s.t.} && \text{(SSP)} \\ &g(\mathbf{Q}_1,\mathbf{Q}_2;\mathbf{Y}_s,\mathbf{Y}_u) \le I && b) \end{aligned}$$

$$\begin{aligned} &\mathrm{Max}\, U(\mathbf{Q}_1,\mathbf{Q}_2;\mathbf{Y}_u) && a) \\ &\text{s.t.} && \text{(DSP)} \\ &h(\mathbf{Q}_1,\mathbf{Q}_2;\mathbf{Y}_u) \le B && b) \end{aligned}$$

where SSP represents the Supply Side of the Problem and DSP the Demand Side of the Problem, given, in congested cases, by the R.T.'s U.M.P..

To date, such type of problems, known as *bilevel* problems, presents many theoretical as well as computational difficulties and represents an active research area so that it is hopeful that most of them will be overcome in the near future. For this reason, in the following we will develop a theoretical framework relating to the demand side of the problem which allows us, first, to formalise in the most general case the R.T.'s U.M. P. which leads to the user equilibrium, and, then, to find out the way to achieve a system equilibrium in a bimodal context.

2. THE DEMAND SIDE OF THE PROBLEM

In order to formalize R.T.'s U.M. P., a brief review of the basic elements of consumer demand theory in the context of *utility maximisation problem* is now helpful. In so doing we will refer to Varian[5] where any detail can be found on the subject.

2.1 Direct and indirect utility function

In the context of utility-maximising behaviour, the consumer is assumed to be faced with possible consumption bundles in his consumption set *X*, and to have preferences on the consumption bundles which satisfy certain standard properties so as to allow to summarise his behaviour by means of a utility function *U*(*x*) which often is a very convenient way to describe consumer's preferences.

Let b be the budget available to a consumer and let $\mathbf{p}$ be the vector of prices of goods. It is assumed that preferences satisfy the hypothesis of local *nonsatiation* so that, for any given $\mathbf{p}$ and b, the utility maximisation problem can be written as:

$$\begin{aligned} &\operatorname*{Max}_{\mathbf{x}} U(\mathbf{x}) \\ &\text{s.t. } \mathbf{px} = b \qquad \mathbf{x} \in X \end{aligned} \tag{1}$$

where there is a *unique* bundle that maximise utility.

Let $\tilde{U}(\mathbf{p}, b) = U^*(\mathbf{x})$ be the solution of problem (1) which, in virtue of the above properties, always exists. $U(\mathbf{x})$ is the *direct* or *unconditional* utility function while $\tilde{U}(\mathbf{p}, b)$ is called the *indirect* or *conditional* utility function. The first one reproduces the consumer's preferences, the second one the utility received after a given choice has actually been made. The function that relates $\mathbf{p}$ and b to the demanded bundle is called the consumer's *demand* function and is denoted by $\mathbf{x}(\mathbf{p}, b)$.

2.2 Aggregating across goods and across consumers

Let the consumption bundle be partitioned into two sub-bundles so that it takes the form $(\mathbf{T}, \mathbf{Z})$ where **T** is the vector of "consumption" of different kinds of trips and **Z** the vector of consumption of *all other goods*. Let the vector **p** be analogously partitioned into $(\mathbf{c}, \mathbf{q})$ where **c** is the price vector of **T** and **q** the price vector of all other goods. Problem (1) can then equivalently be written as

$$\begin{aligned} &\underset{\mathbf{T},\mathbf{Z}}{\text{Max}}\, U(\mathbf{T}, \mathbf{Z}) \\ &\text{s.t.} \\ &\mathbf{cT} + \mathbf{qZ} = b \end{aligned} \qquad (2)$$

In order to make it possible to apply the utility-maximising behaviour to travel demand modelling it should be possible to partition the available income b into b_{T} and b_{Z}, the expenditure on the **Z**-goods and on the **T**-goods, respectively, and to reduce the overall maximisation problem to a sub-utility maximisation problem so as to determine the optimal choice of the **T**-goods, that is to say the different kinds of travels.

To this end let the direct utility function be *separable*, i.e. the consumer's preferences over the **T**-goods are independent of the **z**-goods, which is an actually plausible assumption. The overall utility from **T** and **z** can then be written (McFadden[6]) as a function of the two sub-utilities $u_{\mathrm{T}}(\mathbf{T})$ and $u_{\mathrm{Z}}(\mathbf{Z})$ where $U(u_{\mathrm{T}}(\mathbf{T}), u_{\mathrm{Z}}(\mathbf{Z}))$ is an increasing function of u_{T} and u_{Z}. Moreover, let $T = T(\mathbf{T})$ and $Z = Z(\mathbf{Z})$ be some scalar quantity indices that give the average "amount" of goods consumed, and let $C = C(\mathbf{c})$ and $Q = Q(\mathbf{q})$ be some scalar price indices that give the "average price". Finally, let the direct utility function be homothetic so as to write the expenditure functions as:

$$\begin{aligned} e(\mathbf{c}, u_{\mathrm{T}}) &= e(\mathbf{c}) \cdot u_{\mathrm{T}} \\ e(\mathbf{q}, u_{\mathrm{Z}}) &= e(\mathbf{q}) \cdot u_{\mathrm{Z}} \end{aligned} \qquad (3)$$

By assuming $e(\mathbf{c}) = C$ and $e(\mathbf{q}) = Q$ it is possible to formulate the overall utility maximisation problem at the aggregate level as:

$$\begin{aligned} &\underset{T,Z}{\text{Max}}\, U(T, Z) \\ &\text{s.t.} \\ &CT + QZ = b \end{aligned} \qquad (4)$$

where the price and quantity indices behave like ordinary prices and quantities.

After determining the two total expenditures CT and QZ it is possible to formulate the partial optimisation problem to determine the optimal choice of the **T**-goods as:

$$\underset{\mathbf{T}}{\operatorname{Max}} U(\mathbf{T}, Z)$$
$$\text{s.t.} \qquad (5)$$
$$\mathbf{cT} + b_Z = b$$

where b_Z represent the expenses for all other goods, and whiere Z, b, and b_Z are known quantities. The above result, where all the formulas and variables refer to a single consumer, completely solve, as theoretical framework, our problem of modelling travel demand in the context of utility maximising behaviour on condition that there is no congestion traffic and therefore no external costs imposed on other travellers.

When consumption generates external effects, as in fact it happens in congested networks which we are to deal with, where any individual traveller's behaviour is dependent on and influenced by that of all other travellers, the only way to determine the aggregate demand is at an aggregate level by formulating and solving the R.T.'s U.M.P. This can be done if the aggregate behaviour looks as though it were generated by a single *representative* consumer. It is proved that this is the case on condition that the individual consumer's *indirect* utility functions take the so called **Gorman** form:

$$\tilde{U}_n(\mathbf{c}, b_i) = f_n(\mathbf{c}) + g(\mathbf{c}) \cdot b_n \qquad (6)$$

where the subscript n refers to the nth individual consumer.

As (6) shows, the Gorman form implies that the marginal utility of income, referring to any indirect utility function, is independent of the level of income and constant across consumers. This is a really strong hypothesis which must be taken some care of when modelling urban travel demand, possibly by subdividing the set of interested individuals in an adequate number of sub-sets internally homogeneous.

2.3 The Representative Traveller's Utility Maximisation Problem

In modelling urban travel demand a four-level hierarchical logit model will be used whose correspondent structure of *systematic* utilities are depicted in figure 1 where the origin zone i is assumed given. The four-level decision making concerns: travel or not to travel, subscript t; destination, subscript j; mode, subscript m; route, subscript r.

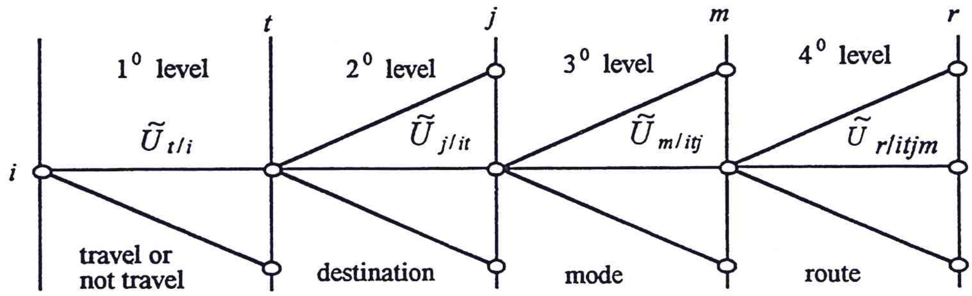

Figure 1 Structure of systematic utilities for a four-level hierarchical choice

With reference to a given origin zone i let $P_{t/i}$, $P_{j/ti}$, $P_{m/tij}$, $P_{r/tijm}$ be individual traveller's probabilities of choosing: whether to travel or not, conditional to zone i; the destination j, conditional to i and t; the mode m, conditional to i, t, and j; the route r, conditional to i, t, j, and m, in that order.

As known (see, for instance Oppenheim[7]) the random utility terms in any indirect utility function are identically and independently Gumbel distributed, each one of the above conditional probabilities, which represents a one-level choice, is given by a logit model, while the *joint* probability that an individual located in origin zone i travels to destination j, on mode m, and following on that mode route r, formally given by:

$$P_{tjmr/i} = P_{t/i} \cdot P_{j/it} \cdot P_{m/itj} \cdot P_{r/itjm} \tag{7}$$

is given by a four-level nested logit model as follows:

$$P_{tjmr/i} = \frac{e^{\beta_1(\tilde{U}_{t/i}+\tilde{W}_{jmr/it})}}{1+e^{\beta_1(\tilde{U}_{t/i}+\tilde{W}_{jmr/it})}} \cdot \frac{e^{\beta_2(\tilde{U}_{j/it}+\tilde{W}_{rm/itj})}}{\sum_j e^{\beta_2(\tilde{U}_{j/it}+\tilde{W}_{rm/itj})}} \cdot \frac{e^{\beta_3(\tilde{U}_{m/itj}+\tilde{W}_{r/itjm})}}{\sum_m e^{\beta_3(\tilde{U}_{m/itj}+\tilde{W}_{r/itjm})}} \cdot \frac{e^{\beta_4 \tilde{U}_{r/itjm}}}{\sum_r e^{\beta_4 \tilde{U}_{r/itjm}}} \tag{8}$$

where each term of the product on the right-hand side in formula (8) represents the corresponding conditional probabilities in formula (7), and where

$$\tilde{W}_{r/itjm} = \frac{1}{\beta_4} \ln \sum_r e^{\beta_4 \tilde{U}_{r/itjm}} \tag{9a}$$

$$\tilde{W}_{mr/itj} = \frac{1}{\beta_3} \ln \sum_m e^{\beta_3(\tilde{U}_{m/itj}+\tilde{W}_{r/itjm})} \tag{9b}$$

$$\tilde{W}_{jmr/it} = \frac{1}{\beta_2} \ln \sum_j e^{\beta_2(\tilde{U}_{j/it}+\tilde{W}_{mr/itj})} \tag{9c}$$

where $\tilde{W}_{x_1,\ldots,x_n/y_1,\ldots y_m}$ represents in any (m+n)-level hierarchical choice process, the *best* an individual traveller chosen at random receives on the average (Anderson et al.[8]) from his jointly repeated choices of $x_1, x_2, \ldots, x_n$ conditional to $y_1, y_2, \ldots, y_m$ having already been chosen.

Formulas (8) and (9), which imply $\beta_1 \leq \beta_2 \leq \beta_3 \leq \beta_4$, are a generalisation of the results obtained in McFadden [6] concerning a two-level hierarchical choice process.

2.3.1 Uncongested case

It is by definition:

$$T_i = N_i \cdot P_{t/i}; \qquad \forall i;\ N_i \text{ given} \tag{10a}$$
$$T_{ij} = T_i \cdot P_{j/it}; \qquad \forall i, j \tag{10b}$$
$$T_{ijm} = T_{ij} \cdot P_{m/itj}; \qquad \forall i, j, m \tag{10c}$$

$$T_{ijmr} = T_{ijm} \cdot P_{r/ijm}; \qquad \forall i, j, m, r \tag{10d}$$

which also imply:

$$T_{ijmr} = T_{ijm} \cdot P_{r/itjm} = T_{ij} \cdot P_{mr/itj} = T_i \cdot P_{jmr/it} = N_i \cdot P_{tjmr/i}; \ \forall i, j, m, r; \ N_i \text{ given} \tag{11}$$

$$\sum_r T_{ijmr} = T_{ijm}; \qquad \forall i, j, m \tag{12a}$$

$$\sum_m T_{ijm} = T_{ij}; \qquad \forall i, j \tag{12b}$$

$$\sum_i T_{ij} = T_j; \qquad \forall j \tag{12c}$$

$$\sum_i T_i = \sum_j T_j = T = \text{total demand travel} \tag{12d}$$

When there is no congestion, the utility functions attached (see figure 1) to any given individual choice are fixed (not influenced by other individual's choices). The above involved probabilities, whose expressions are synthesised in figure 2, can then be evaluated and the aggregate traffic flows determined. In this case, solving the R.T.'s U.M. P., which is an *alternative way* to determine the aggregate demand, is not convenient.

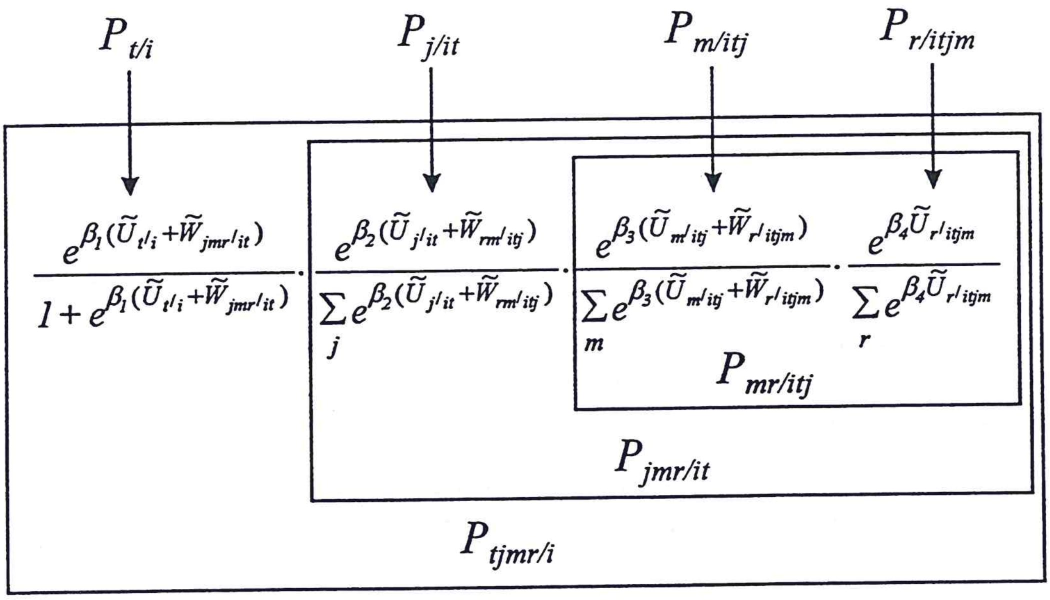

Figure 2. Scheme of joint and conditional probabilities

2.3.2 Congested case

The fundamental difference between the uncongested and the congested case is that the utility functions, specifically those linked to destination and route choice, are now unknown, because affected by the choices of travellers themselves, and are to be determined internally as part of model's solution. Consequently, the *only way* to determine the aggregate demand is at the aggregate level, as solution of the R.T.'s U.M. P., which is no more an *alternative way* as it was in the uncongested case.

Formulating a R.T.'s U.M. P. requires aggregating both across goods and across consumers, which in turn requires, as we have seen in section 2.2, the direct utility functions to be *separable* and the indirect utility functions to be of the *Gorman* form. According to the first hypothesis, which is absolutely plausible, we assume the problem to be like that in formulas (5), while, to take into account the above recommendation linked with the assumption of the second hypothesis, the vector **T** in formula (5), which in the present case of a combined four level travel demand is of the kind $\mathbf{T}\{\{T_i\},\{T_{ij}\},\{T_{ijm}\},\{T_{ijmr}\}\}$, should be thought of as being partitioned into a number of sub-vectors, each component representing a R.T. referring to an internally homogeneous sub-sets of travellers. With reference to one R.T. the R.T.'s U.M. P. can be formally written as:

$$\underset{\mathbf{T}}{Max}\, U(\mathbf{T}; Z) \tag{13}$$

s.t.

$$\sum_{i,j,m,r} c_{ijmr} T_{ijmr} + b_Z = B \tag{14}$$

$$\sum_r T_{ijmr} = T_{ijm}; \qquad \forall i, j, m; \tag{15a}$$

$$\sum_m T_{ijm} = T_{ij}; \qquad \forall i, j,; \tag{15b}$$

$$\sum_{ji} T_{ij} = T_i \qquad \forall i; \tag{15c}$$

where the (15) represent the consisteny constraints.

The problem now is to find out the particular expression to be given the aggregate direct utility function $U(\mathbf{T}, Z)$ in order to retrieve the combined *travel-destination-mode-route* demands conforming to the individual utility maximisation approach. On this regard, it is helpful to recall that, when modelling urban travel demand, the following assumptions are generally made:

- The systematic utility an individual actually receives from making one trip form origin zone i to destination zone j on mode m and route r is formally given by:

$$\tilde{U}_{tjmr/i} = \tilde{U}_{t/i} + \tilde{U}_{j/it} + \tilde{U}_{m/itj} + \tilde{U}_{r/itjm}; \tag{16a}$$

- each term on the right hand side of the (16a) is a linear function of an adequate number of attributes characterising: the origin and the destination zone, the mode, the route, and, ultimately, the individual travellers;
- in the first level utility function, $\tilde{U}_{t/i}$ in the present case, the individual budget, or income, b_i is always included, as an attribute characterising the individual travellers;
- the utility function $\tilde{U}_{r/itjm}$ is generally specified as:

$$\tilde{U}_{r/itjm} = -(c_{ijmr} + \tau \cdot t_{ijmr}) = -g_{ijmr}$$

(17)

where t represents the average travel time and g the average *generalised* cost. The parameter

τ, which represents the conversion coefficient between units of travel time and units of money, is to be estimated by means of calibration.

Leaving unspecified the remaining utility terms because unnecessary at this stage, the (16a) can formally be written as follows:

$$\widetilde{U}_{ijmr/i} = b_i + X_i + X_{ij} + X_{ijm} - \tau \cdot t_{ijmr} - c_{ijmr} = b_i + X_i + X_{ij} + X_{ijm} - g_{ijmr} \tag{17b}$$

where , for short, $\sum_k a_i^k x_i^k = X_i$; $\sum_l a_{ij}^l x_{ij}^l = X_{ij}$; $\sum_s a_{ijm}^s x_{ijm}^s = X_{ijm}$.

Given any *individual indirect* utility function such as the (17b), the corresponding *aggregate direct* utility function of the representative traveller corresponding to aggregate demand referring to a combined travel, mode, destination, and route choice, taking in mind that the t_{ijmr} terms are not fixed, with reference to independent modal networks, can be formally expressed as follows:

$$U_{TDMR} = B_Z - \tau \sum_m \sum_a \int_0^{v_a^m} t_a^m(v)dv - \frac{1}{\beta_4} \sum_{ijmr} T_{ijmr} \ln T_{ijmr} + \sum_{ijm} X_{ijm} T_{ijm} +$$
$$-\left(\frac{1}{\beta_3} - \frac{1}{\beta_4}\right) \sum_{jm} T_{ijm} \ln T_{ijm} + \sum_{ij} X_{ij} T_{ij} - \left(\frac{1}{\beta_2} - \frac{1}{\beta_3}\right) \sum_{ij} T_{ij} \ln T_{ij} + \tag{18}$$
$$+ \sum_i X_i T_i - \left(\frac{1}{\beta_1} - \frac{1}{\beta_2}\right)\left(\sum_i T_i \ln T_i + \sum_i T_{i0} \ln T_{i0}\right)$$

where, for short,

$$v_a^m = \sum_{ijmr} T_{ijmr} \delta_{ijmr}^a \tag{19}$$

and where

$$\delta_{ijmr}^a = \begin{cases} 1; & \text{if the link } a \text{ is part of route } r \text{ between } i \text{ and } j \text{ on mode } m \\ 0; & \text{otherwise} \end{cases} \tag{20}$$

The maximization of the (18) subject to the same constraints (14) and (15), is proved to be a convex problem whose solution retrieves the aggregate demand conforming to nested logit model (see, for the details Oppenheim[7]), 241-250).

2.3.2.1. Generalization to multiple user case

Let us consider the case where the set of all individual travellers is split up into a number N of sub-sets, each internally homogeneous, each represented by a different R.T., and let $\widetilde{U}_{ijmr/i}^n$ and U_{TDMR}^n, $n = 1,2,\ldots,N$, be, the systematic indirect utility conforming to (16b), and the corresponding aggregate direct utility conforming to the (18), respectively, referring to the *nth* subset.

Assuming that the aggregate direct utility function is given by:

$$U_{TDMR} = \sum_{n=1}^{N} U_{TDMR}^{n} \qquad (21)$$

can be with no difficulty proved that the procedures, above recalled, leading to the (18), can be paralleled to the case of N different R.T.s, substituting for the symbol $f(\cdot)$ the symbol $\sum_{n=1}^{N} f^{n}(\cdot)$, in the (18), and the symbol $f^{n}(\cdot)$; $n = 1,2,\ldots,N$, in the (14), (15), (17).

On this basis and taking into account the (18) we can say that, in summary, the *aggregate direct* utility function referring to N different R.T.s consists of:

- an entropy term for each level of the traveller choice process and for each R.T.;
- an aggregate utility term for each individual utility term X_i, X_{ij}, X_{ijm}, and X_{ijmr}, and for each R.T., where the aggregating process results from simple summation or from an adequate integration depending on the utility term in question being fixed or affected by other travellers choices;
- a term B_Z^n for each R.T. denoting the amount spent on nontravel item.

A part from computational and operating difficulties, the two cases, one or more than one different R.T.s, are equivalent and in the following we will refer to the first.

3. TRANSFORMING THE USER EQUILIBRIUM INTO A SYSTEM EQUILIBRIUM

Solving the budgetary constraint for B_Z, substituting in (19) after rearranging the route costs in terms of link costs and deleting B, which, being a constant, does not affect any maximization process, we get:

$$\begin{aligned} S_{TDMR} = & -\tau \sum_{m} \sum_{a} \int_{0}^{v_a^m} g_a^m(v)dv - \frac{1}{\beta_4} \sum_{ijmr} T_{ijmr} \ln T_{ijmr} + \sum_{ijm} X_{ijm} T_{ijm} + \\ & -\left(\frac{1}{\beta_3} - \frac{1}{\beta_4}\right) \sum_{jm} T_{ijm} \ln T_{ijm} + \sum_{ij} X_{ij} T_{ij} - \left(\frac{1}{\beta_2} - \frac{1}{\beta_3}\right) \sum_{ij} T_{ij} \ln T_{ij} + \\ & + \sum_{i} X_i T_i - \left(\frac{1}{\beta_1} - \frac{1}{\beta_2}\right)\left(\sum_{i} T_i \ln T_i + \sum_{i} T_{i0} \ln T_{i0}\right) \end{aligned} \qquad (22)$$

which represents the aggregate *surplus* of the representative traveller under consideration.

The maximization of the (22) subject to the consistency constraints, represents the R.T.'s Surplus Maximization Problem (R.T.'s S.M. P), corresponding to the R.T.'s U.M. P., and retrieves, obviously, the same above aggregate demands conforming to nested logit model.

On the other hand, the traveller surplus may be used as an indicator of social welfare since, *at a market level, transport consumer's surplus reflects the net sum of gains and losses of all* *producers and consumers* (Jara-Diaz and Farah[9]). It follows that the differences between the solution of the R.T.'s S.M. P. and the solution of the above general bilevel problem depend on and are related to the so-called *market failures*. This also means that, to the same extent to which we succeed in both evaluating and reducing the market failures effects, we can also assume the R.T.'s S.M. P. to be a good approximation of the bilevel problem.

With specific reference to urban transport case, and recalling that we are dealing with short run interventions, the main reasons of economic inefficiency are:

1. the external costs imposed on other transport users by individual mode choice decisions, which are especially important in highly congested conditions;
2. the external costs associated with noise, pollution, and accidents;
3. the increasing economies of scale generally characterising the public transport networks with a downwards sloping marginal cost curve, i.e., marginal costs lower than average costs.

As regards equity considerations, which very often is one of the most important concern when making decision about new investments, it is reasonable to assume that it does not play an important role in this context. In fact, it is universally recognised that using transportation to achieve income distribution objectives is not an effective tool: generally weak in the long run interventions, it is certainly insignificant in short run interventions. It would be more effective to use general taxation and social welfare policies to achieve distribution effects and leave the transport market to function at the best.

3.1 Interacting modal link in a bimodal context

The (22) refers to independent modal networks. When dealing with interacting modal links, the first term on the right hand side of formula (22) does not hold any more.

With reference to a bimodal context characterised by cars and buses sharing the same network, let:

$$g_a^m = g_a^m(v_1, v_2); \quad m = 1,2 \qquad (23)$$

be the generalized link cost functions.

It is well known, and easily verified that the (22) still holds on condition that a) the (23) are such that the following condition is met:

$$\frac{\partial g_a^1(v_1, v_2)}{\partial v_2} = \frac{\partial g_a^2(v_1, v_2)}{\partial v_1} \qquad (24)$$

and b) the first term on the right hand side of formula (22) is replaced by:

$$\frac{1}{2}\tau \sum_a \left\{ \int_0^{v_a^1} g_a^1(x, v_2)dx + \int_0^{v_a^2} g_a^2(v_1, x)dx \right\} \qquad (25)$$

The condition (24) is not easily acceptable, because it implies that the effect of an additional unit of modal demand on the other mode's link cost are symmetric. In order to overcome this difficulty, we modify the (23) as follows:

$$\begin{cases} g_a^1 = g_a(v_1 + v_2) + g_a^1(v_1) \\ g_a^1 = g_a(v_1 + v_2) + g_a^2(v_2) \end{cases} \tag{26}$$

where the functions $g_a(v_1 + v_2)$, certainly symmetric, $g_a^1(v_1)$ and $g_a^2(v_2)$ are to be settled on the basis of traffic theory when cars and buses share the same network (see, for instance, Papola[10]).

All we need, in fact, is to determine expressions $g_a^1(v_1, v_2)$ and $g_a^2(v_1, v_2)$ satisfactory from the traffic flow theory stand point, such that the following condition is met:

$$\frac{\partial S_{TDMR}}{\partial v_m} = g_a^m(v_1, v_2); \quad m = 1,2 \tag{27}$$

3.2 Theoretical System Equilibrium (TSE)

In order to remove the above inefficiencies, an integrated urban transport pricing and financing policy is needed so as to achieve a more efficient equilibrium where the costs within the whole transport market are internalised, by raising taxation to the private transport mode, which exhibits an upwards sloping Marginal Cost (MC) curve, and subsidising the transit mode, which exhibits a downwards sloping marginal cost curve. On the grounds of all the above theoretical considerations, we are now in a position to formalize the R.T.'s S.M. P. in socially optimal terms and, ultimately, to find out the way to achieve practically acceptable solutions.

The first step is quite immediate. Given the generalized *average* cost functions $g_a^m(v)$ we can determine the generalized *marginal* travel cost functions $f_a^m(v)$:

$$f_a^m(v_1, v_2) = g_a^m(v_1, v_2) + \frac{\partial g_a^m(v_1, v_2)}{\partial v} \cdot g_a^m(v_1, v_2); \qquad m = 1,2 \tag{28}$$

Substituting for the $g_a^m(v_1, v_2)$ functions the $f_a^m(v_1, v_2)$ in the original R.T.'s S.M.P. we get a System Surplus Maximization Problem (S.S.M.P.), whose solution supplies us with a System Equilibrium (SE), instead of the previous User Equilibrium (UE).

In principle, all we need to achieve such a result, is to price the private car mode and to subsidise the transit mode according with the differences between marginal costs and average costs at the link level. The intervention should not give place to deficit, provided that the difference between the SE and the US consists, substantially, of a modal shift towards the generally less expensive public transport mode.

3.3 Practical system equilibrium

The procedure described in the previous sub-section is unworkable in practice. In fact, given that a complete pricing and financing scheme at link level has been determined, it is patently evident that a toll collection system actually implementing such a scheme is far away from being available. It follows that the precision achieved in modelling and solving the problem is destined to vanish in some measure when implementing the scheme.

To clear up this idea, let us refer to the case , assuming that a toll is levied on all private car entering the inner area **I**, while, as far as the transit mode is concerned, it is assumed that the frequency of service will be increased according to demand increase. The different kind of typical trips on both modes to be taken into consideration are:

1. **A↔I,** from outer area to inner area;
2. **A↔A,** from outer area to outer area;
3. **A↔I↔A,** from outer area to outer area trough the inner area.

Given that the toll is one and the same for every vehicle entering the cordon, the question arises on how to achieve an equilibrium as close as possible to the SE resulting from the solution of the R.T.'s S.M.P.. To this end, let us determine the total amount of the toll revenue R and the weighted mean value of the toll P corresponding to the SE as:

$$P = \sum_{i \in \mathbf{A}, j \in \mathbf{I}, r} T_{ij1r} \cdot \left(f_{ij1r} - g_{ij1r}\right) \Big/ \sum_{i \in \mathbf{A}, j \in \mathbf{I}, r} T_{ij1r} \qquad (29)$$

$$R = \sum_{i \in \mathbf{A}, j \in \mathbf{I}, r} T_{ij1r} \cdot P \qquad (30)$$

There is no doubt that, applying a toll as P to each vehicle entering the cordon, an equilibrium flow will be reached surely different, perhaps notably different, from the above Theoretical System Equilibrium (TSE).

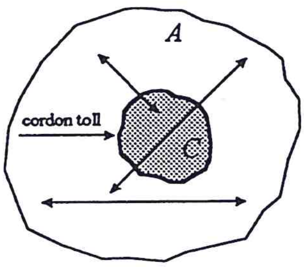

Figure 3. Scheme of different kinds of typical trips when a central area C is surrounded by a cordon toll in order to charge every vehicle entering the cordon

Let $A(i,j,m,r)$ be the set of links a such that $\delta^{a}_{ijmr} = 1$ and let $A(\mathbf{I})$ be the set of all the links lying inside the cordon toll. When a toll P is charged to each vehicle entering the cordon, the generalized perceived cost to car mode will be, in general:

$$g_{ij1r} = \sum_{a} g_a \delta^{a}_{ij1r} + \gamma_{ij1r} \cdot P \qquad (31)$$

where:

$$\gamma_{ij1r} = \begin{cases} 1 & if \quad A\{i,j,1,r\} \cap A\{\mathrm{I}\} \neq \varnothing \\ 0 & if \quad A\{i,j,1,r\} \cap A\{\mathrm{I}\} = \varnothing \end{cases} \tag{32}$$

Substituting for g_{ij1r} the (31) and solving the R.T.'s S.M.P. formalized in terms of disaggregate variables, we can evaluate the difference, in terms of Social Surplus (SS), between the solution so found and the TSE. By repeatedly applying such a procedure for different values for the toll P, we can get the PSE as the solution of the R.T.'s S.M.P approximating at the best the TSE.

4. FURTHER RESEARCH

The paper has provided a modelling framework to address pricing and financing schemes from both theoretical and practical standpoints. The work in progress relates to the development of solution procedures of the problem formulated in terms of aggregate as well as disaggregate variables.

REFERENCES

[1] HILLMAN, M., ET AL. (1973) *Personal Mobility and Transport Policy*, PEP Broadsheet 542, London.

[2] PLOWDEN, S. (1973) *Towns Against Traffic*, Andre Deutsch, London.

[3] SCHAEFFER, K.H. & E. SCLAR (1975) *Access for All*, Penguin, London.

[4] GOODWIN, P.B. (1990) Demographic impacts, social consequences, and the transport debate, *Oxford Review of Economic Policy* **6**(2), 76-90.

[5] VARIAN, H. (1992) *Microeconomic Analysis.* 3rd ed. Norton, New York, 96-102, 144-154

[6] MCFADDEN, D. (1981) Econometric models of probabilistic choice. In Manski, C. and D. McFadden (eds), *Structural Analysis of discrete Data with Econometric Applications.* Harvard University Press, Cambridge.

[7] OPPENHEIM, N. (1995) *Urban Travel Demand Modelling.* Wiley-Interscience, New York.

[8] ANDERSON, S., A. DE PALMA AND J.F. THISSE (1992) *Discrete Choice Theory of Product Differentiation.* MIT Press, Cambridge.

[9] JARA-DIAZ, S.R. AND M. FARAH (1988) Valuation of user's benefits in transport systems. *Transport Reviews* **8**(3), 197-218.

[10] PAPOLA, N. (1989) Uso promiscuo della via (Interacting modes). In Papola, N. and F. Accattatis, *Teoria del Deflusso Veicolare e Suo Impiego nella Progettazione e nella Regolazione* (*Theory of Traffic Flow and Its Use in Planning and Controlling*) parte II, Atti di trasporto, Dipartimento Idraulica, Trasporti e Strade, Roma, 92-106.