COMPETITION FOR BUS AND TRAIN SERVICES: PRODUCT DIFFERENTIATION AND MIXED DUOPOLY*

Pedro Cantos-Sánchez and Rafael Moner-Colonques.
Departamento de Análisis Económico and Laboratorio Integrado de Sistemas Inteligentes de Infomación Aplidados al Tráfico (LISITT), Universidad de Valencia.

1. INTRODUCTION

The spatial nature of the transportation product, service quality and large scale economies stand among the most important features of transportation[1] . There are a number of theoretical models by now which incorporate these characteristics into them to analyse demand and supply determinants, optimal pricing or the desirability of regulating the transportation industries. The latter fact has led economists to take institutional elements into account if any policy implications are to be drawn out. One possible approach is to compare several competition regimes but this is normally done for a particular means of transport (see Evans (1987), Dodgson and Katsoulacos (1988) and Morrison and Winston (1985), only to mention a few).

However, and if we move in an interurban route, the modes of transport compete with each other. A recent exception is Ireland (1991) where consumers' travelling can be done by car or by bus. Here we will be concerned with competition between bus and train transport and the demand side of our model can be viewed as an extension of Ireland (1991). The public enterprise nature of the train company leads us to compare two duopoly regimes with the social optimum. These two regimes are the standard private duopoly and the mixed duopoly, this meaning a public firm -the train company- competing with a private firm -the bus company. The analysis involves different definitions of the objective functions under each scenario and this modelling offers an alternative way to market regulation. Differently, the move from a mixed to a private duopoly, or viceversa, can be looked at in the context of the privatisation/nationalisation debate. There are some justifications to think that a duopoly is the appropriate market structure on the following grounds. On the one hand, a regulated market might explain why there is only one private bus company operating a regular interurban route that competes with the train company. Such would be the case of Spain and Portugal. On the other, a duopolistic structure may be due to imperfect contestability reasons, as in Britain.

Evans (1987) and Ireland (1991) have developed models for bus transport in which the regulated and deregulated scenarios are identified with monopoly and free-entry competition, respectively. Any assessment of deregulation costs and benefits still requires economic models for interpretation of any available evidence. Public intervention in the form of a public firm, whose purpose differs markedly from the usual profit maximisation objective, is another possibility of market regulation. Our paper is a first step to introduce mixed duopoly considerations in a transport economics model.

Despite observed situations in which public and private firms compete within the same industry, the literature on such mixed oligopolies has only recently been elaborated. The issue has become more relevant in the light of the recent debates over privatisation of public firms. The existing contributions have studied mixed oligopolies assuming that firms sell in a

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[1] Winston (1985) surveys the economics of transportation and identifies these and other features as distinctive of transportation economics.

homogeneous good market or in a differentiated products market and the interest of the analysis lies in whether the presence of a public firm is socially desirable; in other words, whether public intervention in this way can lead to a welfare improvement and possibly recover the socially optimal solution[2] .

More specifically, we posit a model which combines horizontal and vertical differentiation aspects. Though arbitrary, we assume that because of travelling time, comfort, reliability, etc..., travelling by train is preferred to travelling by bus for all consumers if services are offered at the same price. In this sense, train transport is vertically differentiated from bus transport and, given that we are interested in both firms being present in the market, the latter has to be cheaper to be chosen by consumers. Once the choice between train and bus has taken place, consumers decide on which service to take. The horizontal differentiation feature refers to the fact that each train service competes with its nearest rivals, respectively for bus services. We are interested in the subgame perfect equilibrium of a two-stage game in which first firms simultaneously choose the frequency of service and then simultaneously choose prices. We will characterise the private duopoly equilibrium, where both the train and bus companies maximise profits, the mixed duopoly equilibrium, where the bus company maximises profits and the train company maximises total surplus, and the socially optimal solution.

One major problem with the introduction of product differentiation is that the analysis gets substantially complicated. However some conclusions within each means of transport can be obtained. A numerical solution is provided that allows us to compare across both means of transport. The main findings are that the bus transport sets a higher frequency of services when it maximises profits compared to the socially optimal solution, while the train transport sets a lower frequency whether it maximises profits or total surplus. The presence of a public firm pushes the private firm to increase the number of services. When the vertical characteristic is relaxed total surplus increases and market shares get closer. It also supposes a redistribution between producer and consumer surplus depending on the chosen scenario.

The remainder of the paper is organised as follows. We begin by presenting the theoretical model. Section 3 is devoted to the characterisation of the mixed duopoly, private duopoly and social optimum equilibria. Then in section 4 we establish some comparison and we provide a numerical example to complement the analysis. A brief concluding section closes the paper.

2. DESCRIPTION OF THE MODEL

We will use a model which combines horizontal and vertical differentiation aspects. Our analysis, following Ireland (1991), takes a sequential view of market decision-making. First, consumers choose whether to travel by train or by bus, and this decision is not related to the conduct of any particular train or bus route. It is, in a sense, a long run decision and may be explained because of the uncertainty in the duration of travelling time or because timetables may change. Then, and given the consumers' choice of the means of transport, the decision on which service to take is made. In other words, we assume that each train service competes for customers with its nearest rivals, respectively for bus services, and that train transport as a whole competes with bus transport as a whole[3].

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[2] De Fraja and Delbono (1989) and Cremer et al. (1989) study mixed oligopolies with homogeneous products, while Cremer et al. (1991) analyse horizontal product differentiation and Grilo (1994) analyses vertical product differentiation.

[3] The combination of horizontal and vertical differentiation in a single model is not easy. On the one hand, the assumption on the sequentiality of market decision-making greatly simplifies the analysis. On the other, it does not seem too unrealistic that consumers take decisions on travelling on this basis.

Individuals have income y; the values of y vary among consumers and are defined by a uniform unit density on (0,1). Potential consumers have a different most-preferred travelling time and this can be represented by a point on a unit length circumference. Consumers, or equivalently consumers' addresses, are uniformly distributed over the circle with a unit density and this density is independent of consumers' incomes. They consume a single unit (that is, one journey) irrespective of its price. There is a cost or disutility associated with the discrepancy between a consumer's 'ideal' travelling time and the scheduled timetable. Then, each consumer decides first which means of transport he/she will use. If prices and frequencies are such that a consumer obtains a negative (expected) utility, he will not travel. Finally, and once he has chosen either the train or the bus transport, he must decide which service he will take. If the travelling is done by train, the consumer chooses the service which maximises,

$$U_t(y) = y - p_t - v d_{it} \tag{1}$$

where y is the consumer's income, p_{it} is the price charged by the ith train service, d_{it} is the difference in time of the ith service from the consumer's ideal departure time, and v is the associated cost to such inconvenience. Given the expected average prices and frequencies set by the train company, the consumer will expect an interval between services of $1/n_t$, where n stands for the frequency. We will use subscripts t and b to refer to train and bus respectively throughout the paper. The symmetry in the model means that, on average, the consumer will bear the cost associated to $1/4n_t$. Since all the train services will be priced equally, equation (1) becomes,

$$E[U_t(y)] = y - p_t - v/(4\, n_t) \tag{2}$$

A completely parallel procedure allows us to write the (expected) utility from bus transport as,

$$E[U_b(y)] = h\,(y - p_b - v/(4\, n_b)) \tag{3}$$

where p_b denotes the (expected) price of bus journeys. The parameter h, with $h<1$, captures a quality differential between both means of transport.We have arbitrarily chosen the train transport to be of a higher quality than the bus transport, that is, if $p_t + v/(4\, n_t) = p_b + v/(4\, n_b)$ all individuals would prefer to travel by train[4].

We now proceed to allocate demands for each means of transport. By equating (2) and (3) we find the income level of the individual indifferent between travelling by train and by bus. We denote it by y^* and is given by,

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They choose the bus, the train or the plane, and then the service that better suits them. The interested reader may refer to Ireland (1987), or to Neven and Thisse (1988) and Anderson et al. (1992) for a different approach to multi-characteristics competition. More recently, see Dos Santos and Thisse (1996).

[4] A straight combination of a standard utility function used to model horizontal and vertical product differentiation would lead, in our case, to a utility function of the form, $EU_b = y + \theta\, Q_b - p_b - v/4\, n_t$, where Q_b is interpreted as the quality of the bus transport, and $\theta \in (\theta_., \theta^.)$, $\theta_. > 0$, is consumer θ's marginal willlingness to pay for quality. The formulation here follows Ireland (1991) and is qualitatively equivalent. It allows more tractability and takes into account the demand effects associated with external competition, coming from the vertical product differentiation, and internal competition, coming from the horizontal product differentiation.

$$y^* = \frac{p_t - h p_b + (v / (4n_t)) - (v h / (4n_b))}{(1 - h)} \tag{4}$$

Demand for train transport will take place for those individuals whose income belongs to the interval $(y^*, 1)$. Then,

$$D_t = \frac{1 - h - p_t + h p_b - v / (4 n_t) + v h / (4 n_b)}{(1 - h)} \tag{5}$$

To determine the number of individuals deciding to use bus transport note that an individual will not travel at all if he gets a negative (expected) utility, i.e. (3)<0. Denote this income level by y^+. This means that the demand for bus transport is given by,

$$D_b = \frac{p_t + v / (4n_t) - p_b - v / (4n_b)}{1 - h} \tag{6}$$

This presentation allows for the market not being completely covered. Note that if $p_t + v/(4\ n_t) < p_b + v/(4\ n_b)$ only the train company would be present in the market. Since we are interested in those cases when both firms are operating, we assume that the inequality does not hold. Furthermore, it eliminates existence problems[5]. The system (5)-(6) defines a model of product differentiation with asymmetric demands. As we will see later on, clearcut results are difficult to obtain because comparisons get rather complicated.

Turning to the horizontal differentiation aspects, a consumer travelling by train will prefer service i to service i+1 if,

$$p_{it} + v\ d_{it} < p_{i+1,t} + v\ ((1/n_t) - d_{it})$$

so that the indifferent consumer has a 'mismatch',

$$d_{it}^* = (p_{i+1,t} - p_{it} + (v/n_t))/2v$$

The demand for service i is given by,

$$D_{ib} = 2\, d_{it}^*\, D_t = \frac{(p_{i+1,t} - p_{it} + (v / n_t))\ (1 - h - p_t + h p_b - v / (4n_t) + v h / (4n_b))}{v\ (1 - h)}$$

and it is composed of the number of consumers travelling by train multiplied by the proportion then opting for that particular service. It is easy to see, given the symmetry of the model $p_{i+1,t} = p_{it}$, that (9) is (5) divided by n_t. A similar argument applies to bus transport. Let us define now the costs of providing train and bus services. It is common to assume a cost c per passenger and a fixed cost $f(n)$ per service. Usually, costs per passenger represent a very small share of the total costs. We will assume that they are zero, i.e. c=0. Then,

$$C_l = f\,(n_l) \qquad l = t,\ b.$$

[5] A price equilibrium in the original Hotelling model may fail to exist because profits are not quasiconcave in own price (see d'Aspremont et al. (1979)).

It is also assumed that $f(0) = 0$, $f(\infty) = \infty$, $f'(n) > 0$, $f''(n) \leq 0$.

3. CHARACTERISATION OF THE EQUILIBRIUM.

We are interested in the subgame perfect equilibrium of the following two-stage game. In the first stage duopolists choose simultaneously the frequency of services, n_t and n_b. In the second stage, firms choose simultaneously prices, p_t and p_b. The model is solved in the standard backward fashion. We will characterise the private and the mixed duopoly cases and then the social optimum equilibrium. The difference between the former two cases lies, following the literature on mixed oligopolies, in the definition of the objective functions for the train and bus companies. These definitions shall shortly be made explicit.

3.1 The Mixed Duopoly Model

In this case the bus company is identified with the private firm and hence it maximises profits. On the other hand, the train company will maximise total surplus, that is, consumer surplus plus profits of both companies. In the literature, total surplus is maximised under the constraint of non-negative profits. As long as prices are positive it will be the case that the public firm remains operative[6]. We start by solving the second stage of the game when firms simultaneously choose prices. The bus company maximises,

$$\underset{p_b}{Max}\, \Pi_b = p_b\, D_b - f(n_b) \tag{11}$$

where D_b is given by (6). In order to write the objective function of the public firm we start by writing the consumer surplus, CS.

$$CS = \int_{y^*}^{1} (y - p_t - v/(4\,n_t)) f(y)\, dy + \int_{y^-}^{y^*} h\,(y - p_b - v/(4\,n_b))\, f(y)\, dy + \int_{0}^{y^-} y f(y)\, dy \tag{12}$$

It includes three terms corresponding to those consumers travelling by train, those travelling by bus and those who do not travel at all, respectively. The train company maximises total surplus, TS_t,

$$\underset{p_t}{max}\, TS_t = CS + \Pi_t + \Pi_b \tag{13}$$

where $\Pi_t = p_t D_t - f(n_t)$. The first order conditions of (11) and (13) read,

6 Alternatively, we could have assumed that the density of consumers over the circle was not unitary, say S for instance. This parameterisation would ensure that the public firm does not violate the non-negativity constraint on profits.

$$p_b = \frac{n_b(4\, n_t\, p_t + v) - n_t\, v}{8\, n_b\, n_t} \tag{14}$$

$$p_t = p_b$$

Though combining horizontal and vertical differentiation, it is the latter aspect that is relevant in the analysis for it takes into account the competition between the two modes of transport. In a mixed duopoly with vertical differentiation, Grilo (1994) shows that the equilibrium configuration is characterised by prices net of marginal cost being identical. In her model marginal costs are dependent on quality. In fact, such condition ensures that the splitting of consumers is optimal when both firms are present on the market. Since from the point of view of the public firm, prices are just transfers between consumers anf firms, she can enforce the optimal assignment of consumers between the two firms by setting a price that satisfies the aforementioned condition. Given that it is the vertical differentiation aspect the one that prevails in our model, it is not surprising that the equilibrium prices set by both firms are equal[7].

The second order conditions are satisfied and do not suppose any additional requirements. Note also that prices are strategic complements, so that the price set by the train company increases in response to increases in the price of the bus transport. The solution to (14) is $p_t^* = p_b^* = \frac{v(n_b - n_t)}{4 n_b\, n_t}$. One point should be noted by looking at the equilibrium prices: for them to be positive it must be the case that $n_b > n_t$. In order to solve for the first stage equilibrium, the values of p_t and p_b are substituted back in (11) and (13) and firms maximise over n_t and n_b, respectively. The first order conditions when the train company chooses n_t and the bus company chooses n_b becomes,

$$\frac{v^2\,(n_b - n_t)}{8\, n_b^3\, n_t\, (h - 1)} + f'(n_b) \tag{15}$$

$$\frac{v\,(2\, h\, n_b\, (2\, n_t - v) + n_b\, (3\, v - 4\, n_t) - n_t\, v)}{16\, n_b\, n_t^3\, (h - 1)} - f'(n_t) \tag{16}$$

The equilibrium number of frequencies cannot be written in an easy and clear way. It is yet easy to see that frequencies are strategic substitutes.. Note that equations (15) and (16) are expressions of reaction functions in implicit form. The second order conditions for a maximum require N to be negative semidefinite, where N_{11} is a matrix the elements of which are $N_{11} = \partial^2 \Pi_b / \partial n_b^2$, $N_{12} = \partial^2 \Pi_b / \partial n_b n_t$,, $N_{21} = \partial^2 TS_t / \partial n_t n_b$,, $N_{22} = \partial^2 TS_t / \partial n_t^2$ .These expressions are relegated to an Appendix.

3.2 The Private Duopoly Model

This is the standard duopoly case in which both firms, the train and the bus companies, maximise profits. The comparison of the equilibrium here with the one obtained in the previous subsection can give us some intuition for what would happen were the public firm privatised. Alternatively, the movement from a private to a mixed duopoly can be read as a nationalisation

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[7] This is of course an analitycal and normative result which is difficult to find in reality. Note that if we had not assumed zero marginal costs per passenger, the interpretation of $p_t = p_b$ would be closer to Grilo's and more in accordance with reality.

strategy. In any case, this is a useful way to evaluate the impact of a public firm in the context of competition between two means of transport.

The train company maximises $\Pi_t = p_t . D_t - f(n_t)$ and the bus company maximises (11). The system of first order conditions is given by,

$$p_b = \frac{n_b (4 n_t p_t + v) - n_t v}{8 n_b n_t} \tag{17}$$

$$p_t = \frac{h (n_t (4 n_b (p_b - 1) + v) + n_b (4 n_t - v)}{8 n_b n_t} \tag{18}$$

The second order conditions for a maximum are satisfied and the solution to (17) and (18) yields,

$$p_b = \frac{h n_t (4 n_b - v) - n_b (4 n_t + v) + 2 n_t v}{4 n_b n_t (h - 4)} \tag{19}$$

$$p_t = \frac{h (n_b (8 n_t - v) - n_t v) - 2 n_b (4 n_t - v)}{4 n_b n_t (h - 4)} \tag{20}$$

As before and solving backwards, the equilibrium prices p_b^+ and p_t^* are substituted in the profits expressions to obtain the equilibrium values of the frequencies set by the duopolists. The first order conditions are given by,

$$\frac{v (h n_t (4 n_b - v) - n_b (4 n_t + v) + 2 n_t v) (h - 2)}{8 n_b^3 n_t (h - 1) (h - 4)^2} + f'(n_b) = 0 \tag{21}$$

$$\frac{v (h (n_b (8 n_t - v) - n_t v) - 2 n_b (4 n_t - v)) (h - 2)}{8 n_b n_t^3 (h - 1) (h - 4)^2} + f'(n_t) = 0 \tag{22}$$

and again frequencies are strategic substitutes. We may characterise the second order conditons as above. The expressions are given in the Appendix.

3.3 The Social Optimum

We define the social optimum as the solution to the maximisation of total surplus, that is, equation (13). In this model it is defined by a pair of frequencies and a pair of prices. In order to characterise it we take the derivative of *TS* with respect to p_b and p_t .The solution obtained is that $p_b = p_t$. It means that, for any given pair of frequencies, the price equilibrium splits the consumers between both means of transport in an optimal way. We have noted above the interpretation of this result. As Grilo (1994) points out, there are two underlying assumptions for it to happen. Our model incorporates both these assumptions: i) consumer surplus and producer surplus are given the same weight in the objective function and ii) there is no quantity effect since all consumers travel one journey. In other words, the equilibrium price configuration obtained in the mixed duopoly case will be socially optimal whereas the one in the private duopoly case will not. Having noted that, we will take $p_t^* = p_b^* = \frac{v (n_b - n_t)}{4 n_b n_t}$ and substitute these values in TS to solve for the equilibrium number of frequencies. The pair of first order conditions follows.

$$\frac{v^2 (n_b - n_t)}{16 n_b^3 n_t (h - 1)} + f'(n_b) \tag{23}$$

$$\frac{v (2 h n_b (2 n_t - v) + n_b (3 v - 4 n_t) - n_t v)}{16 n_b n_t^3 (h - 1)} - f'(n_t) \tag{24}$$

It is easy to see that (24) coincides with (16). This is what motivates the comparison and what stresses the role played by the presence of a public firm in this context.

4. COMPARISON OF EQUILIBRIA

Despite the simplicity of the model we cannot explicitly solve for the equilibrium pair of number of services under each scenario. Yet something can be said about the ranking of frequencies within the bus and train transport, respectively, by comparing the first order conditions. We have grouped them all in table (1) an we use upperscripts *MD*, *PD* and *SO* to denote the mixed duopoly, the private duopoly and the social optimum, respectively.

We start by comparing the mixed duopoly with the social optimum cases for the bus transport. It is straightforward to see that $C_b^{MD} > C_b^{SO}$ and $C_t^{MD} = C_t^{SO}$. This, together with the fact that reaction functions in the frequency space slope downwards means that $n_b^{SO} < n_b^{MD}$ and $n_t^{SO} > n_t^{MD}$. This result is displayed in Figure one. Such a conclusive result cannot be given when we compare with the private duopoly case. However,

If $[4 n_b n_t (h^2 - 3 h + 2) - n_b v (h^2 - 7 h + 14) - 4 n_t v (h - 3)] > 0$, then $C_b^{PD} > C_b^{MD}$, and if $[4 n_b n_t (h^3 - 5 h^2 + 12 h - 8) - n_b v (2 h^3 - 17 h^2 + 48 h - 40) - n_t v (3 h^2 - 12 h + 16)] < 0$, then $C_t^{MD} > C_t^{PD}$. Consequently, $n_t^{SO} > n_t^{MD} > n_t^{PD}$, $n_b^{PD} > n_b^{MD} > n_b^{SO}$. This ranking of equilibrium frequencies is illustrated in Figure two.

But the ranking depends on the parameter values. These outcomes confirm the following points: i) the public firm reaches the highest n_t in the social optimum and , ii) the bus company in the private duopoly case sets a frequency n_b in excess of the social optimum. In the mixed duopoly case it is the lowest quality firm that increases its frequency choice compared to the social optimum.

Concerning prices, it can be checked that in the mixed duopoly case they will be higher compared to the social optimum situation. This is so since $n_b^{MD} > n_b^{SO}$ and $n_t^{SO} > n_t^{MD}$ and given that the price configuration is the same under both cases. No precise answer can be given about the prices in the private duopoly. In any case, whenever p_t under private duopoly is higher than under mixed duopoly, it will be the case that p_b in the private duopoly exceeds both the prices under mixed duopoly and the social optimum.

Given the difficulty in extracting further conclusions from the general model we have resorted to a numerical example. In doing so, let us make some simplifications. Assume that the cost function is linear, that is,$C_t = f_t\, n_t$ and $C_b = f_b\, n_b$, and we take f_t=0.05 and f_b=0.003. These absolute values are of course arbitrary, but they want to capture the relative cost difference between both means of transport. The parameter v is an estimation of the value of the headway (time between services). Some works have estimated it to be important compared to the valuation 'in vehicle time'. The former effect is relatively lower than the latter effect for an interurban route, the opposite happens for an urban route (see Fowkes and Nash (1991) and Matas (1991), respectively). We take v=0.075. Finally, the parameter h collects service quality aspects such as travelling time (probably the most important aspect), comfort, reliability, etc., and we provide an example with different values for h. Table (2) displays the results. We have checked that for these parameter values the second order conditions are satisfied. The range of values for h is rather small in the example since other values imply either the non-existence of our two-stage equilibria or the violation of the second order conditions.

Bus	MD	$\frac{v^2 (n_b - n_t)}{8 n_b^3 n_t (h-1)} + f'(n_b) = C_b^{MD}$
	PD	$\frac{v (h n_t (4 n_b - v) - n_b (4 n_t + v) + 2 n_t v)(h-2)}{8 n_b^3 n_t (h-1)(h-4)^2} + f'(n_b) = C_b^{PD}$
	SO	$\frac{v^2 (n_b - n_t)}{16 n_b^3 n_t (h-1)} + f'(n_b) = C_b^{SO}$
Train	MD	$\frac{v (2 h n_b (2 n_t - v) + n_b (3v - 4 n_t) - n_t v)}{16 n_b n_t^3 (h-1)} - f'(n_t) = C_t^{MD}$
	PD	$\frac{v (h (n_b (8 n_t - v) - n_t v) - 2 n_b (4 n_t - v))(h-2)}{8 n_b n_t^3 (h-1)(h-4)^2} + f'(n_t) = C_t^{PD}$
	SO	$\frac{v (2 h n_b (2 n_t - v) + n_b (3v - 4 n_t) - n_t v)}{16 n_b n_t^3 (h-1)} - f'(n_t) = C_t^{SO}$

Table 1. The first order conditions for the frequency of services.

	h= 0.8975			*h*=0.9			*h*=0.91		
	MD	*PD*	*SO*	*MD*	*PD*	*SO*	*MD*	*PD*	*SO*
n_b	1.8353	1.3920	0.9457	1.8801	1.3977	1.0220	2.1020	1.4240	1.2296
n_t	0.4955	0.3670	0.5435	0.4901	0.3654	0.5348	0.4603	0.3582	0.5064
p_b	0.0276	0.0447	0.0146	0.0282	0.0440	0.0167	0.0318	0.0414	0.0217
p_t	0.0276	0.0518	0.0146	0.0282	0.0502	0.0167	0.0318	0.0436	0.0217
D_b %	0.2694	0.4362	0.1431	0.2828	0.4404	0.1671	0.3534	0.4602	0.2419
D_t %	0.6927	0.5055	0.8223	0.6789	0.5021	0.7978	0.6058	0.4852	0.7210
Π_b	0.0019	0.0153	-0.0007	0.0023	0.0152	-0.0002	0.0049	0.0147	0.0015
Π_t	-0.0056	0.0078	-0.0151	-0.0053	0.0069	-0.0130	-0.0037	0.0032	-0.0096
EC	0.4407	0.4128	0.4536	0.4400	0.4140	0.4514	0.4360	0.4187	0.4460
ET	0.4370	0.4360	0.4377	0.4371	0.4361	0.4378	0.4372	0.4368	0.4379

Table 2. A numerical example.

A few comments are in order. The bus transport sets a higher frequency of services when it maximises profits compared to the socially optimal solution, while the train transport sets a lower frequency whether it maximises profits or total surplus. The presence of a public firm pushes the private firm to increase the number of services. Although this is a perverse effect, it is true that consumer surplus increases because prices are lower and, on aggregate, total surplus is higher. This is obviously an argument in favour of a public firm. Logically, an assessment about the desirability of a privatisation strategy must take into account the efficiency gains derived by cost reductions. They should compensate for the welfare loss in the private duopoly case.

Table (2) also shows the effect of a variation in the parameter h. As it increases, that is, the degree of vertical differentiation shrinks, the frequency of bus services increases while the number of train services decreases. Total surplus increases due to a profits increase that more than compensates the consumer loss given that prices, when h tends to unity, are larger. This is what happens in the mixed duopoly and social optimum cases. However, competition is more intense in the private duopoly case and prices are lower the bigger the h. The quantity effect (frequencies) dominates the reduction in prices which leads to a reduction in profits of the bus company and a gain for consumers. At the same time, the market shares of both modes of transport get closer. Finally, the gain in total surplus is relatively more marked than the gains in the mixed duopoly and the social optimum cases. Therefore, an increase in market competition as a result e.g. of a reduction in the bus travelling time (bigger h) leads to a redistribution between producer and consumer surpluses.

Had we assumed a concave cost function for the train transport, the frequency for this means of transport would have increased compared to the linear case. This is easily understood since a concave cost function for the train company would be collecting the presence of increasing returns to scale. Hence, strategic intercation between the two modes of transport results in a lower frequency of bus services and an increase in total surplus given that the high quality mode captures a bigger market share.

5. CONCLUDING REMARKS

We have developed a model in which there is competition between the train and the bus services. The model combines horizontal and vertical differentiation aspects. Once the demands for each means of transport has been determined, we solve for the subgame perfect equilibrium under several scenarios of a two-stage game: firms simultaneously choose the frequency of services and then they set prices.

Despite the simplicity of the model, expressions become rather unmanageable in trying to assess the impact of a public firm (the rail company). We may conclude by means of a numerical example that the bus transport sets a higher frequency of services when it maximises profits compared to the socially optimal solution, while the train transport sets a lower frequency whether it maximises profits or total surplus. The presence of a public firm pushes the private firm to increase the number of services. When the vertical characteristic is relaxed total surplus increases and market shares get closer. A final remark is that a mixed duopoly achieves welfare levels that are higher than under a private duopoly and closer to the socially optimal solution.

We have used quite a restrictive setting and any policy recommendations should be taken with the necessary qualifications. In this sense, Cremer et al. (1991) find that the presence of a public firm may have a positive effect on overall welfare depending on the market structure. Also, it is typically the case that market regulation not only restricts access to the market, as we noted in the introduction, but it also imposes price or frequency controls. The reference point to such degree of intervention could be taken to be the social optimum case. Then, the movement to a mixed or to a private duopoly can be read as partial deregulating mechanisms. Having said that, the model offers some suggestive conclusions applicable to some current situations and related with variations in the quality parameter. Thus, the reduction in bus travelling time because of a better motorway infrastructure has led to an increase in the service frequency and therefore to a greater presence of bus companies in the market. In fact, this has happened in most of the Spanish interurban routes. Another related example is an improvement in the rail services as indicated by the introduction of high speed rail services. Their use has supposed an increase in the number of passengers and frequency of services. Of course, a welfare evaluation is difficult to assess since several modes of transport are competing with each other and infrastructure costs must be considered (see De Rus, 1993, for a welfare evaluation of the Spanish AVE Madrid-Sevilla).

Figure 1

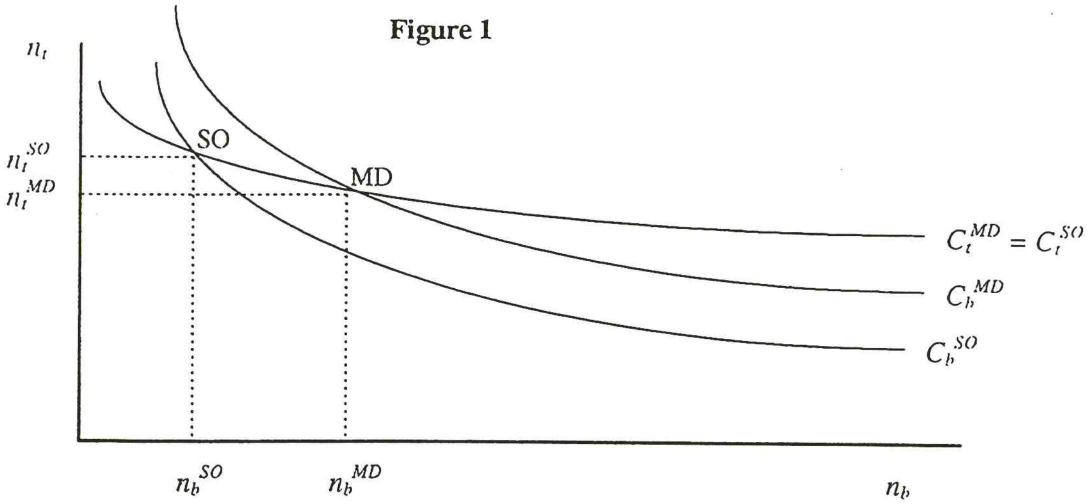
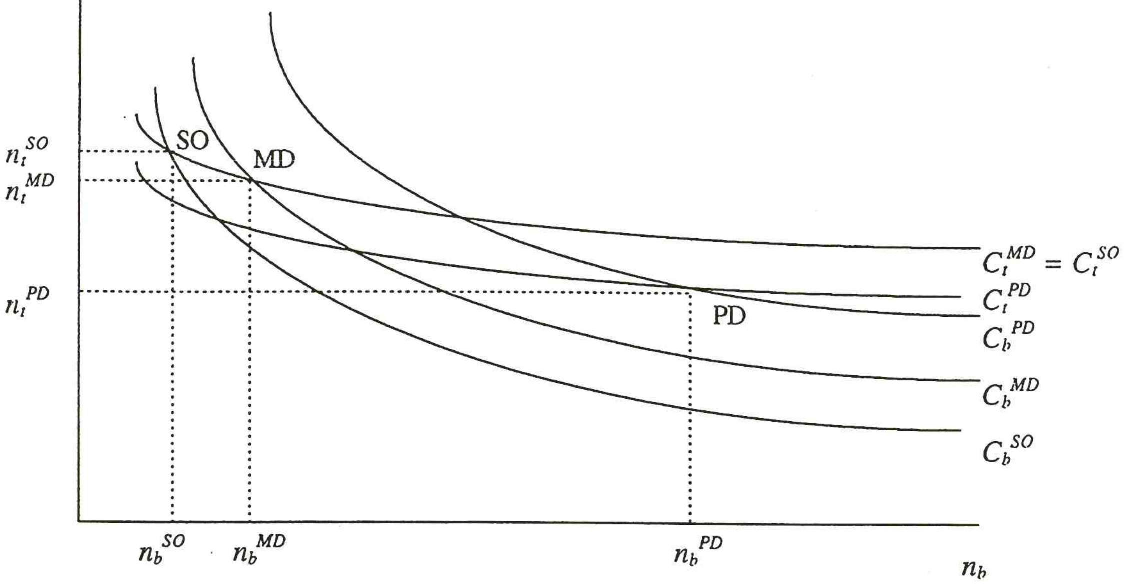


Figure 2



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7. APPENDIX

These are the derivatives to compute the second order conditions in:

Mixed Duopoly Model.

$$N_{11} = \frac{\partial^2 \Pi_b}{\partial n_b^2} = \frac{v^2 (2 n_b - 3 n_t)}{8 n_b^4 n_t (h - 1)} - f''(n_b)$$

$$N_{12} = \frac{\partial^2 \Pi_b}{\partial n_b n_t} = \frac{v^2}{8 n_b^2 n_t^2 (h-1)}$$

$$N_{21} = \frac{\partial^2 ET}{\partial n_t n_b} = \frac{v^2}{16 n_b^2 n_t^2 (h-1)}$$

$$N_{22} = -\frac{v (2 h n_b (2 n_t - v) + n_b (3 v - 4 n_t) - n_t v)}{16 n_b n_t^4 (h - 1)} - f''(n_t)$$

Private Duopoly Model.

$$N_{11} = \frac{\partial^2 \Pi_b}{\partial n_b^2} = \frac{v (h n_t (8 n_b - 3v) - 2 (n_b (4 n_t + v) - 3 n_t v)) (h-2)}{8 n_b^4 n_t (h-1)(h-4)^2} - f''(n_b)$$

$$N_{12} = \frac{\partial^2 \Pi_b}{\partial n_b n_t} = \frac{v^2 (h-2)}{8 n_b^2 n_t^2 (h-1)(h-4)^2}$$

$$N_{21} = \frac{\partial^2 \Pi_t}{\partial n_t n_b} = \frac{v^2}{16 n_b^2 n_t^2 (h-1)}$$

$$N_{22} = \frac{\partial^2 \Pi_t}{\partial n_t^2} = \frac{v (h (n_b (16 n_t - 3v) - 2 n_t v) - 2 n_b (8 n_t - 3v)) (h-2)}{8 n_b n_t^4 (h-1)(h-4)^2} - f''(n_t)$$

Social Optimum.

$$\frac{\partial^2 ET}{\partial n_b^2} = \frac{v^2 (2 n_b - 3 n_t)}{16 n_b^4 n_t (h-1)} - f_b''(n_b) < 0$$

$$\frac{\partial^2 ET}{\partial n_t^2} = \frac{-v (2h (n_b (4 n_t - 3v) + n_b (9v - 8 n_t) - 2 n_t v)}{16 n_b n_t^4 (h-1)} - f_t''(n_t) < 0$$