Design and Evaluation of Transit Routes in Urban Networks

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Abstract

This paper describes the transit network design problem and proposes a new approach taking account of both passenger and operator interests. It is explained that although the redesign of existing transit networks is not an activity that should often be undertaken by transit properties; as an infrequent initiative it can have significant impacts on transit system performance.

The method described in this paper is used to create, analyze, and optimize, transit route networks. The developed system creates all feasible routes and transfers that connect every place (node) on the network to all others. Out of this vast pool of routes and transfers, it generates smaller subsets, each maintaining connectivity on the network. For each of the subsets generated, the system fulfills transportation demand by calculating the appropriate frequency for each route. Next, it calculates the optimization parameters for each subset:

(i) Passenger Hours (defined as passengers' riding time in a bus on an hourly basis. It measures how much time is spent by passengers on buses).

(ii) Waiting Time (defined as the amount of time passengers spend waiting at a bus stop).

(iii) Empty Passenger Hours (defined as the unused seats in a bus on an hourly basis. It measures to what capacity buses are used).

(iv) Fleet Size (number of buses needed to provide all trips along the chosen set of routes).

The system is also designed as a software tool called **TROPT** (**T**ransit **R**oute **OPT**imization). As such, the user is able to choose the most suitable subset based on the specific optimization parameter he desires. The system has been designed as a tool to plan future transit networks as well as to help maintain existing ones. This flexibility is achieved throughout the system by allowing the user to supply either his own data or to execute it automatically.

The approach proposed in this paper is intended to be easier to implement and more sensitive to the risks of making route design changes than previous methods and current practice.

Introduction

The problem addressed in this paper is a general one of how to design a new bus network or redesign an existing bus network given no a-priori specifications as to the desired network structure. From a practical perspective, it is desirable that the route design procedures include interaction and feedback loops between the selection of effective routes and the operational scheduling components: Frequency determination and time-tabling along with vehicle scheduling with regard to fleet size.

The system generates all feasible routes and transfers that connect every place (node) in the network to all others. From this vast pool of possible routes and transfers, it then generates smaller subsets, which maintain network connectivity. For each subset thus generated, the system meets transportation demands by calculating the appropriate frequency for each route. Next, it calculates pre-specified optimization parameters for each subset. Based on the specific optimization parameter desired by the user, it is then possible to select the most suitable subset. The system has been designed as a tool for the planning of future transit networks as well as the maintenance of existing ones. The system ensures flexibility by allowing the user to either input his own data or to run the system automatically.

Problem Identification

The transit planning process, aimed at efficient transport of origin-destination transit-riders, includes four basic components performed in sequence: (a) Network Route Design; (b) Setting Timetables; (c) Scheduling Vehicles to Trips; and (d) Assignment of Drivers. For this process to be cost-effective and efficient, it should embody a compromise between passenger comfort and cost of service. For example, a good match between bus supply and passenger demand occurs when bus schedules are constructed so that the observed passenger demand is accommodated while the number of vehicles use is minimized.

Ceder and Wilson (1986) discuss the four components of the process and emphasize that the bulk of resources is devoted to the last two steps: bus and driver scheduling. In North America, these components are generally referred to as vehicle blocking (a block is a sequence of revenue and nonrevenue activities for an individual bus) and driver run-cutting (splitting and recombining vehicle blocks into legal driver shifts or runs). This concern for the cumbersome and time-consuming manual scheduling of components is reflected in many papers and numerous computer programs designed to automate these steps (at least partially). At the same time, only a few researchers have studied the interrelationship between the scheduling components and the network design element. The interrelationship exists in two directions: (i) each set of routes yields, based on the demand, a different set of frequencies and timetables and, ultimately, the required fleet size, and (ii) the operational cost derived from the scheduling components and the passenger level of service affect the search for the optimal route design while relying on a compromise between the operator and the user.

Practical network design focuses almost entirely on individual routes that have been identified as candidates for change. However, it may be possible that the overall bus network could be improved through restructuring of the entire network. For many North American properties that have not been reappraised in this respect since the 1940's, it is high time to consider precisely such an undertaking. Such considerations motivated the present research to seek an efficient network route design method based on certain objective functions and a set of constraints. The main purpose of the method is to transport a given origin-destination demand through the bus network in the most cost-effective way. The special characteristics of route design problems are: (a) Passenger demand is spread throughout the entire network where it is generated and terminated at many points along the network's links and can be grouped in terms of an origin-destination matrix; (b) The demand is to be transported simultaneously; (c) Over a given planning horizon, it is impossible to reconstruct the routes; i.e., once the route network is designed, it will remain as it is over an entire planning period.

Prior approaches to these issues can be grouped into those which simulate passenger flows, those which deal with ideal networks and those based on mathematical programming. A similar but different perspective and review of prior approaches can be found in Ceder and Wilson (1986). Simulation models are presented in Dial and Bunyan (1968), Rapp et al., (1976), Heathington et al., (1968), and Vandebona and Richardson (1985). These models require a considerable amount of data, and their proximity to optimality is uncertain. Ideal Network methods are based on a broad range of design parameters and a choice of objectives reflecting user and operator interest. Such methods appear in Kocur and Henrickson (1982), Tsao and Schonfeld (1984), and Kuah and Perl (1988). These methods are adequate for screening or policy analyses in which approximate design parameters are to be determined rather than a complete design. Thus, these methods cannot represent real situations. Mathematical Programming models used for transit network design are inevitably heuristic due to the extremely high computational effort required by them. These partial optimization approaches appear in Hasselstrom (1981), Dubois et al., (1979), Lampkin and Saalmans (1967), Silman et al. (1974), Rea (1971), Mandl (1980), Marwah et al. (1984), Sharp (1974), and Keudel (1988). Apart from Hasselstrom's model, which is included in the Volvo transit planning package, all other models have not been actually applied. However, the Hasslestrom model is quite complex, non-user oriented and expensive both in terms of the data required and the direct cost and staff time needed for the process. The disadvantages of the existing mathematical programming models can be summarized in six points:

(i) Cannot handle large size transit networks;

(ii) Do not consider operational objectives and constraints;

(iii) Vehicle frequency determination is based on economic parameters rather than on passenger counts (as is done in practice and described by Ceder [1984, 1986]);

(iv) Cannot incorporate simultaneously three out of the four planning components: network design, setting timetables, and vehicle scheduling. In particular, the models cannot evaluate the network without defining the vehicle requirements for each route and thus lacks precision in evaluating the cost effectiveness of design;

(v) Cannot produce good results consistently;

(vi) Cannot incorporate non-quantitative constraints such as imposing certain links to be included in transit routes and considering operational strategies.

In this paper, an alternative mathematical programming approach is presented which aims to be more practical, given typical data availability, and less complex than other models. This will allow for increasing the chances of acceptance within most transit properties.

Research Approach

The approach presented in the present paper combines the philosophy of the mathematical programming approaches with decision-making techniques, so as to allow the user to select from a number of alternatives. The methodology of this study is schematically presented in Figure 1.

The transit route design problem is based on two objective functions Z_1 and Z_2:

$$\min\ z_1=\alpha_1\sum_{i,j\varepsilon N} PH(i,j)+\alpha_2\sum_{i,j\varepsilon N} WH(i,j)+\alpha_3 \sum_{r} EH_r \qquad (1)$$

$$\min\ z_2=FS \qquad (2)$$

where,

PH(i,j) = Passenger Hours between nodes i and j, i, j ε N (defined as passengers' riding time in a bus on an hourly basis. It measures how much time is spent by passengers on buses between the two nodes);

WH(i,j) = Waiting Time between nodes i and j, i, j ε N (defined as the amount of time in a bus on an hourly basis. It measures how much time is spent by passengers on buses between the two nodes);

EH_r = Empty Space-Hours on route r (defined as the unused seats in a bus on an hourly basis. Empty Space-Hours measures to what capacity buses are used);

FS = Fleet Size (number of buses needed to provide all trips along the chosen set of routes);

The complete formulation, including the constraints of the network design problem can be found in Israeli (1990). The nature of the overall formulation is non-linear (non-linear and mixed integer programming). Its analog problem is the generalized network design problem described by Magnanti and Wong (1984) with an NP-hard computational complexity. Thus, conventional approaches are incapable of providing a solution even with a relatively high degree of simplification.

In the first stage, the problem dimension is reduced through the construction of a skeleton feasible route network that meets a maximum travel time constraint. The skeleton network is the basis for an optimization routine to determine the shortest direct and indirect (via transfers) paths between each pair of nodes. The second stage relies on a procedure that incorporates optimization and enumeration processes to derive the minimal Z_1 objective function. This procedure, while searching for min Z_1, creates various Z_2 solutions — each associated with a different Z_2 solution. Finally, the most desirable set of (Z_1, Z_2) is derived through known techniques in multiobjective programming. (Cohon [1978], Goicoechea et al. [1982]). Further details of the theoretical dimension of this methodology can be found in Israeli (1990).

Components:

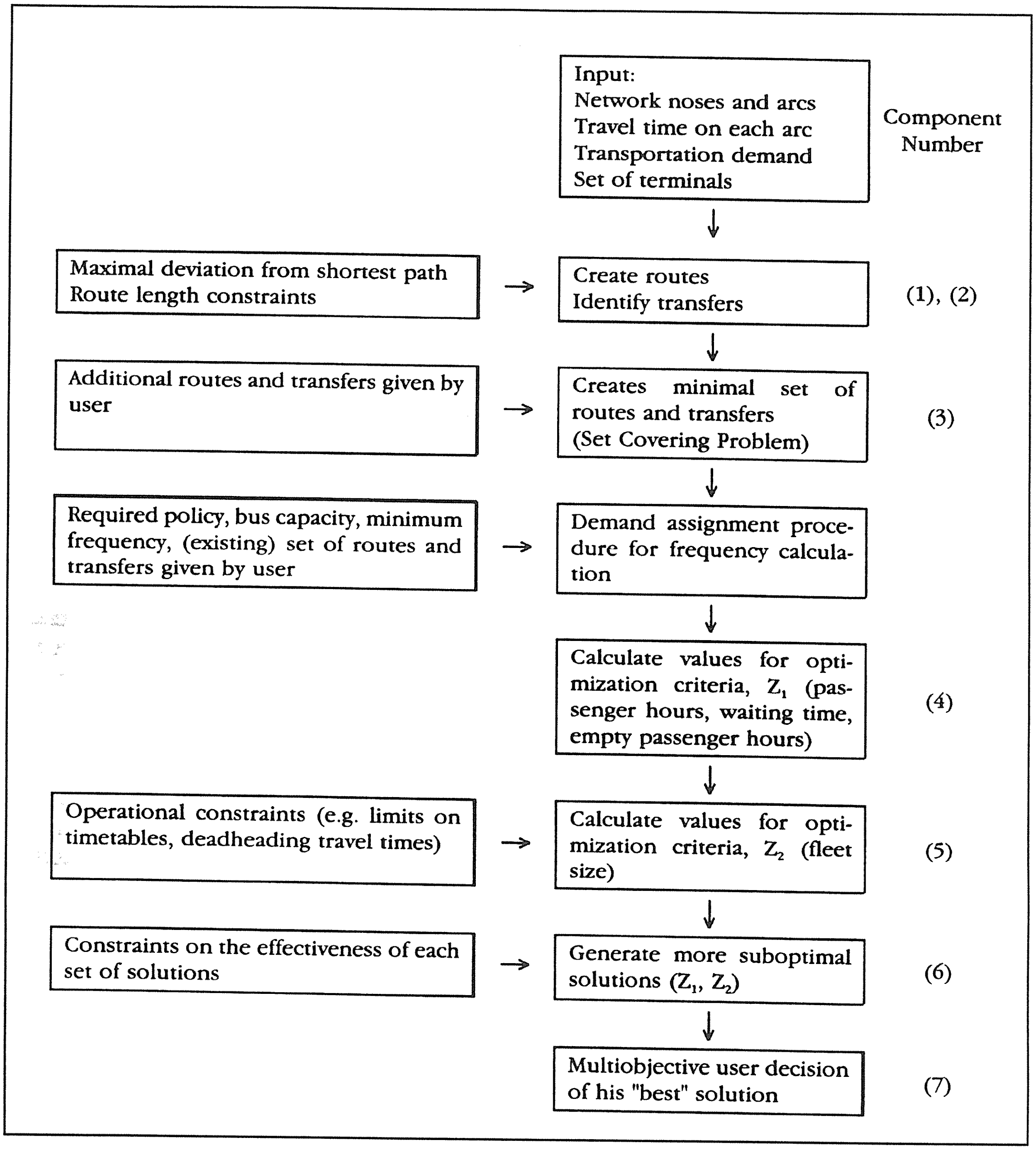
The overall system comprises seven components (as shown in Figure 1). **In the first component**, the system generates every possible route from all terminals, which meets the route length factor constraints. In other words, its algorithm screens out routes according to given boundaries on the route length. In addition, there is a limit on the travel time between each origin-destination (O-D) pair. That is, a given demand cannot be assigned to a bus route if its travel time exceeds the minimum travel time on the network by more than a given percentage (100α). This constraint is similar to the criterion used by Ceder and Wilson (1986). The outcome of this part is a set of routes connecting only some of the O-D pairs. The disconnected O-D pairs are then subjected to transfers. In this component the user is able to introduce his own routes, (either existing or manually planned routes).

A simple 8-node example is used throughout the paper as an expeditory device to demonstrate the procedures used. The basic network with two terminals (from which trips can be initiated) is shown in Figure 2 with the input demand presented in Table 1.

The outcome of the first component is presented in Table 2, while using $\alpha = 0.4$ (no route length or its portion can exceed its associated shortest travel time by more than 40 percent).

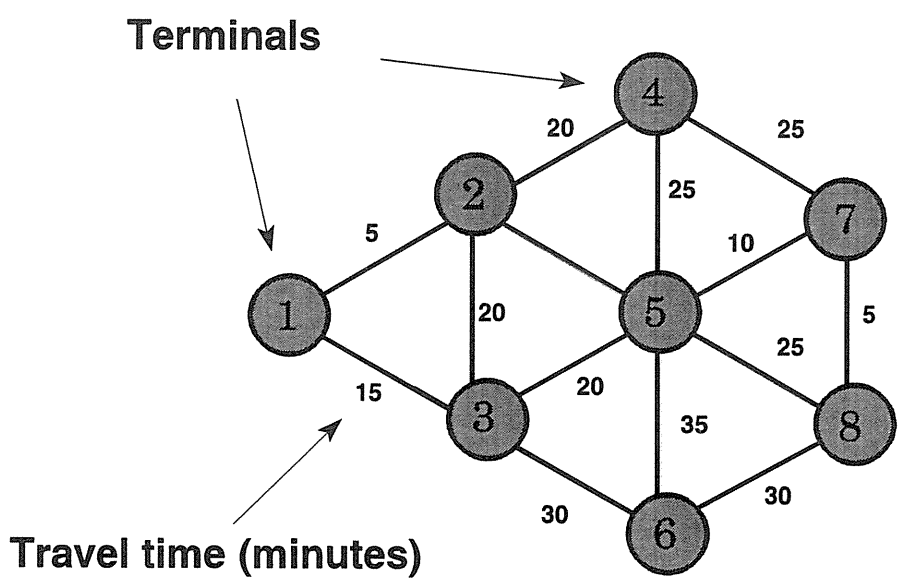
The second component is based on an algorithm that mainly produces feasible transfers throughout the entire network. The first step of the algorithm is to establish additional direct routes between O-D pairs characterized by high O-D demands (a predetermined O-D demand value). These direct routes are actually initiated and/or terminated at non-terminal nodes, and, consequently, deadheading trips are responsible for their connection to the terminals. Also, a low O-D demand without a direct route is not considered for obtaining service. The transfers are created using a mapping algorithm. This applies to the disconnected

Figure 1: Schematic Flow Chart of the Methodology



O-D pairs as well as to all the O-D pairs. The algorithm detailed in Israeli (1990) considers the maximal allowed degree of transfers within the bus network. The degree of transfers is defined as the number of routes that are involved in one O-D trip minus one. In practice, its maximal value is one or two. An O-D transfer is created subject to the additional limit on its travel time, (it cannot be greater than a certain percentage over the shortest travel time). If for an O-D demand, none of the transfers are feasible, then the least violating one is chosen. The algorithm is based on recurrence relation — initially, it searches for a first degree transfer such that its travel time is within the given limit and then continues with the next degree of transfers. The outcome of this module is a set of routes for each O-D pair that describes the path of the transfer and the transfer's nodes. At this stage there is a large pool of routes and transfers, all of which provide connectivity throughout the network and meet the travel time and RLF constraints.

Figure 2: Example of an 8-node Network with Its Basic Input



Route diversion (from shortest path) factor (α) = 0.40
Maximum degree of transfers = 2
Bus capacity = 50

Table 1

The Demand between Nodes for the Example Problem (assumed to be symmetrical)

	1	2	3	4	5	6	7	8
1	0	80	70	160	50	200	120	60
2	80	0	120	90	100	70	250	70
3	70	120	0	180	150	120	30	250
4	160	90	180	0	80	210	170	230
5	50	100	150	80	0	250	40	130
6	200	70	120	210	250	0	130	120
7	120	250	30	170	40	130	0	70
8	60	70	250	230	130	120	120	0

However, the large set of routes and transfers is likely to contain many overlapping segments (of routes and transfers). An overlapped segment is one that serves O-D pairs that are completely served by other routes and/or transfers. An overlapped route comprises segments that are all totally overlapping. The latter is treated in the next component.

Table 2

All Routes of the Example Problem Generated by the First Component

Route #	Description
1	1 → 2
2	1 → 2 → 4
3	1 → 2 → 5
4	1 → 2 → 5 → 6
5	1 → 2 → 5 → 7
6	1 → 2 → 5 → 7 → 8
7	1 → 2 → 5 → 7 → 8 → 6
8	1 → 3
9	1 → 3 → 6
10	4 → 2
11	4 → 2 → 1 → 3
12	4 → 2 → 1 → 3 → 6
13	4 → 2 → 3
14	4 → 2 → 3 → 6
15	4 → 2 → 5
16	4 → 2 → 5 → 6
17	4 → 5
18	4 → 5 → 3
19	4 → 5 → 6
20	4 → 5 → 7
21	4 → 5 → 7 → 8
22	4 → 5 → 7 → 8 → 6
23	4 → 7
24	4 → 7 → 5
25	4 → 7 → 5 → 3
26	4 → 7 → 5 → 6
27	4 → 7 → 8
28	4 → 7 → 8 → 6

In the example problem there are 18 transfers, which meet the criteria. These transfers are shown in Table 3 where the numbers in parentheses are the route numbers (see Table 2) that comprise the transfers. In the transfer path description, the numbers outside the parentheses represent nodes, while those inside the parentheses represent all routes.

In the third component, the system creates minimal set(s) of routes and their related transfers such that

Table 3

All Routes of the Example Problem Generated by the First Component		
Transfer #		**Description**
29	(5, 18, 27)	3(18) → 5(5) → 7(27) → 8
30	(5, 18, 28)	3(18) → 5(5) → 7(28) → 8
31	(6, 18)	3(18) → 5(6) → 7(6) → 8
32	(6, 25)	3(25) → 5(25,6) → 7(6) → 8
33	(7, 18)	3(18) → 5(7) → 7(7) → 8
34	(7, 25)	3(25) → 5(25,7) → 7(7) → 8
35	(18, 20, 27)	3(18) → 5(20) → 7(27) → 8
36	(18, 20, 28)	3(18) → 5(20) → 7(28) → 8
37	(18, 21)	3(18) → 5(21) → 7(21) → 8
38	(18, 22)	3(18) → 5(22) → 7(22) → 8
39	(18, 24, 27)	3(18) → 5(24) → 7(27) → 8
40	(18, 24, 28)	3(18) → 5(24) → 7(28) → 8
41	(18, 26, 27)	3(18) → 5(26) → 7(27) → 8
42	(18, 26, 28)	3(18) → 5(26) → 7(28) → 8
43	(21, 25)	3(25) → 5(25,21) → 7(21) → 8
44	(22, 25)	3(25) → 5(25,22) → 7(22) → 8
45	(25, 27)	3(25) → 5(25) → 7(27) → 8
46	(25, 28)	3(25) → 5(25) → 7(28) → 8

connectivity between nodes is maintained and their total deviation from the shortest path is minimized. This problem is defined as a Set Covering Problem (SCP), which is hard to solve (see Minieka (1978), Syslo et al. [1983]). The SCP can determine the minimal set of routes from the matrix of the feasible routes. In this matrix each row represents either a feasible route or a transfer. The "1" in the matrix is inserted whenever an O-D demand can be transported by the route or transfer and "0" otherwise. The word "covering" refers here to at least one column with "1" in each row. The transfers are combined columns in the SCP matrix and therefore increase the complexity of the problem. No solution appears for this problem in the literature.

In this work, an algorithm has been developed and tested with a random network. Its solution was compared with: (i) integer programming optimization without considering transfers; (ii) with non-linear programming using relaxation methods on the integer variables for bus networks with transfers; and (iii) with a complete enumeration of all possible covering scenarios. The outcome of this module is a set of the minimum number of routes that cover all the O-D pairs in the network. Israeli (1990) provides a heuristic search algorithm that includes three covering variations:

a) Covering only the shortest paths between all O-D pairs;

b) Covering all the O-D paths while minimizing the number of paths that are not the shortest;

c) Covering all the O-D paths while minimizing the combined paths' travel time.

Nine sets of routes are generated in the example problem. These sets are presented in Table 4 and refer to the routes and transfers indicated in Tables 2 and 3.

In the fourth component, the entire O-D demand is assigned to the chosen set of routes. The assignment algorithm that has been developed includes steps that are related to a route-choice decision investigation; i.e., the algorithm includes a probabilistic function for passengers who are able to select the bus that arrives first or, alternatively, wait for a faster bus. The passengers' strategy is to minimize the in-vehicle travel time. The methodology used is similar to that developed by Marguier and Ceder (1984) but with a different probabilistic function. This function considers the route length and the length of a transfer path between each O-D pair, where both the routes and transfer paths are divided into "slow" and "fast" categories. The bus frequency is used in the function as a variable and is derived from the passenger load profile of each route. The load on each route is determined by the demand and assignment method. The heart of the whole module is a set of equations (3rd order degree), which are solved by an iterative method developed especially for this problem. The outcome of the module is: bus frequencies of the set of routes, passenger load profiles, demand assignment across the set of routes, and the optimization parameters PH, WH, EH for computing Z_1.

The fifth component completely represents the operator's perspective. It is directed at finding the minimum fleet size required to meet the demand as well as to satisfy the determined frequency on each route. This objective is designated Z_2, in the formulation, and it complements the operator's portion in Z_1, which is to minimize empty space-hours.

The method used for evaluating the fleet size is based on the Deficit Function theory proposed by Ceder and Stern (1981). This theory provides techniques to assign the minimum number of vehicles to carry out a given timetable. A deficit function is simply a step function, which increases by one at the time of each trip departure and decreases by one at the time of each trip arrival. Such a function may be constructed for each terminal in a multi-terminal transit system. The sum of the maximal deficit function values over the schedule horizon and across all the terminals is the minimum number of vehicles required. The maximal value of the deficit function can be reduced by introducing deadheading (empty) trips into the schedule as well as shifting the departure times within bounded tolerances as described by Ceder and Stern (1982). This study does not include all the detailed procedures of the deficit function theory and concentrates, rather, on estimating the minimum fleet size required for a fixed schedule (shifting of departure times is not allowed). For this purpose, the lower bound calculation described in Stern and Ceder (1983) is used, based on the determined frequencies in the fourth module of the system. The final outcome of this is the value Z_2, which represents the operational component's minimum required fleet size.

The sixth component is responsible for constructing alternative sets of routes to search for additional (Z_1, Z_2) values in the vicinity of their optimal setting. The procedure for this search is based on incremental changes in the set of routes, much like the reduced gradient methods. Given the set of routes associated with the minimum Z_1 value, the single route that is the worst contributor to Z_1 is deleted and then the SCP

Table 4

All Subsets Generated in the Example Problem	
Set #	Description
1.	{4, 6, 9, 11, 25, 28, 32, 46}
2.	{7, 9, 11, 19, 25, 27, 34, 45}
3.	{7, 9, 11, 25, 28, 34, 46}
4.	{6, 9, 11, 16, 25, 28, 32, 46}
5.	{6, 12, 19, 25, 28, 32, 46}
6.	{7, 12, 25, 27, 34, 45}
7.	{7, 12, 25, 28, 34, 46}
8.	{4, 6, 12, 25, 28, 32, 46}
9.	{6, 12, 16, 25, 28, 32, 46}

is solved in the third module, followed by the execution of the fourth and fifth components. This process could continue, but there is no guarantee that a previous alternative will not be repeated. To overcome this problem, a new matrix is constructed with the idea of finding the minimal and worst set of candidate routes for possible deletion in each iteration; i.e., a new SCP matrix is constructed in which the candidate routes are the columns, and each row represents a previous set of routes, which was already identified in the vicinity of the optimal (Z_1, Z_2) setting. The solution to this new SCP matrix is a set of rejected routes so as not to repeat a previous alternative solution. During this process, a number of unique collections of routes are termed "prohibited columns" as they are the only ones which can transport a certain demand. These prohibited columns are assigned an artificially high cost value, so as not to be included in the solution. This process also involves some bounds to converge on a desired number of iterations, or number of (Z_1, Z_2) solutions.

In the example problem, we set $\alpha_1 = \alpha_2 = \alpha_3 = 1$ in the objective function, Eq. (1), for the weighted sum of the optimization parameters and the lower bound on the fleet size in Eq. (2). Nine "good" sets were produced by the sixth component as shown in Table 4, and their (Z_1, Z_2) values appear in Table 5.

The seventh and final component of the system involves multi-objective programming of the two objective functions Z_1 and Z_2. Given the alternative sets of routes derived in the sixth module, the purpose is to investigate the various alternatives regarding the most efficient (Z_1, Z_2) solution. The method selected in this component is called the compromise set method based on Zeleny (1973, 1974). It fits linear objective functions from which Duckstein and Opricovic (1980) derived solutions for discrete variables. The outcome of this method is the theoretical point in which (Z_1, Z_2) attains its relatively minimal value. The results can be presented in a table or a two-dimensional graph that shows the trade-off between Z_1 and Z_2. The decision-maker can then decide whether or not to accept the proposed solution. In the latter case, for example, he can see how much Z_1 is increased by decreasing Z_2 to a certain value and vice versa.

The trade-off situation regarding the example problem is depicted in Figure 3. The lower left corner of

Table 5

Alternative Sets of Routes in the Example Problem

SETS #	Z_1	$Z_2 = FS$
1	788	106
2	900	109
3	1105	117
4	866	102
5	937	103
6	997	101
7	1213	113
8	869	103
9	961	105

the envelope contour of the nine solutions represents the best sets. The user is then able to choose his desired solution. (In the example case, the choice is between [Z_1 = 788, Z_2 = 106] and [Z_1 = 866, Z_2 = 102]).

Summary

This paper outlines the basics of TROPT (**T**ransit **R**oute **OPT**imization) - a transit network design system. The overall objective of TROPT is to create, analyze and optimize bus route networks. The system generates all feasible routes and transfers which connect every place (node) on the network with all others. From this vast pool of possible routes and transfers it generates smaller subsets, each maintaining connectivity on the network. For each of the subsets generated the system meets transportation demands by calculating the appropriate frequency for each route, and determines pre-specified optimization parameters for each subset.

The optimization parameters used are:

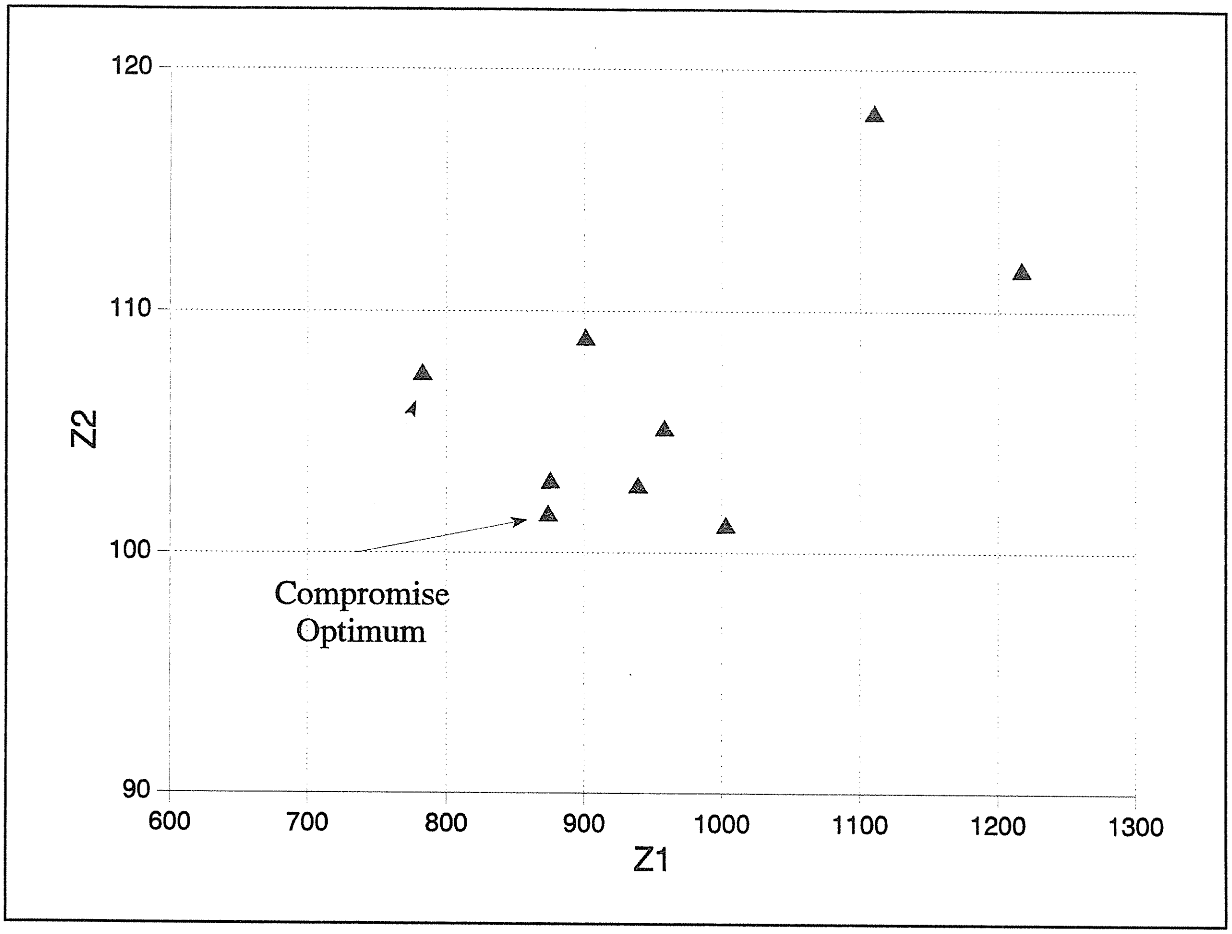
(1) Empty Passenger Hours (defined as the unused seats in a bus on an hourly basis. The parameter of Empty Passenger Hours measures to what capacity buses are used);

(2) Passenger Hours (defined as passengers' riding time in a bus on an hourly basis. This parameter measures how much time is spent by passengers on buses);

(3) Waiting Time (defined as the amount of time passengers spend waiting at a bus stop);

(4) Fleet Size (number of buses needed to provide all trips along the chosen set of routes);

Figure 3: Trade-off between Z_1 and the Minimal Fleet Size, Z_2, in the Example Problem.



The system comprises seven main components, each of which uses the data created by its predecessor (or data given by the user) to carry on the next step in the analysis (see Figure 1). TROPT is believed to be a useful toolset for the following applications:

(a) Optimal design for a new transit network.

(b) Optimal design for expansion or curtailment of an existing transit network.

(c) Assessment of the performance of an existing transit network from the aspects of: (i) Operator efficiency. (ii) Passenger level of service.

(d) Sensitivity analysis of transit network performance for a variety of system parameters (such as different bus fleet size, different level of service, changes in passenger demand, changes in frequencies, changes in travel time and more).

Finally, it is worth mentioning that the challenge of TROPT, which was ultimately met, was to incorporate three operational planning components simultaneously at the macro and micro levels: Network Design, Frequency (Timetable) Setting, and Vehicle Scheduling.

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