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# PRICING AND FINANCING OF THE RAILWAY IN A COMPETITIVE ENVIRONMENT*

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## 1 INTRODUCTION

Much concern has been dedicated to rail services, both from the general public and politicians in most countries. Should rail services be state-owned or private? Should they be supported financially? Are rail services financially and socially viable or are they obsolete? Paradoxically there are two simultaneous trends, rail lines are shut down and high-speed tracks and trains are introduced. Financially no entire national railway system is profitable, especially if infrastructure costs are taken into account, even though certain lines may be very successful even from financial point of view.

During the last two decades another trend has flourished: deregulation. In Western Europe this trend commenced within local and regional public transport. The privatisation of the English bus industry, even the long distance coach services, represents the "full market solution", where both supply, prices and the operation are in the hands of competing profit maximising firms. In the Nordic countries, and to some extent in the US and France, the decision over local and regional public transport supply and prices has bee kept in the hands of a public authority, while the actual operation is left for competition through tendering. Typically these services need local or central government grants for financing – and there are economic rationales for this.

Rail transport has in most countries so far been left in the hands of government controlled bodies. In England the railway is split into a rail track company and operators, all privatised. In Sweden the Swedish Rail (SJ) has been split into a social welfare oriented Railway Administration (Banverket) responsible for infrastructure investments (financed by the government) and the "commercial" new SJ, with the aim to operate the service at a minimum profit determined by the government. SJ still enjoys monopoly for the commercially viable lines for passenger transport, while the non-viable ones are put out for tender by a new government authority (Rikstrafiken). With respect to freight transport there is free competition “on the track”. The reader should keep in mind that the author of this paper basically has the Swedish organisational form in mind, where the rail service operators are

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commercial, but most of the analysis is supposed to be general since welfare and profit oriented regimes are contrasted.

This paper does not aim to solve or even discuss all the issues related to rail transport. One aim is to analyse whether there may be economic reasons for subsidisation of passenger railway services through consumer prices or infrastructure charges. The second aim is to analyse these subsidisation issues under second-best conditions. In this thus important to contrast welfare and profit maximising policies in terms of prices and supply, since many railway companies now act commercially. We will also briefly discuss whether vertical integration or separation between track and operational responsibilities may be of some significance in this context and whether competition "on the track" seems justifiable.

The model considers two principally different cases:

- A welfare maximising authority determines simultaneously price, service frequency and number of carriages per train. This case corresponds to vertical integration under a government welfare maximising monopoly.

- Profit maximising operators determine simultaneously price, service frequency and number of carriages per train. In this case a welfare maximising rail track authority determines infrastructure charges and consumer subsidies. This case corresponds to vertical separation with independent profit maximising operators and a supervising welfare maximising authority.

Joint determination of price and supply of public transport from a welfare point of view was first presented by Mohring [1972]. The approach has then been followed by e.g. Turvey and Mohring [1975] and J. O. Jansson [1979], [1984] who deal with price and service frequency, using models which are most relevant for *frequent urban services* and assuming one passenger group. Nash [1978] optimises price and output in terms of miles operated for *frequent urban bus services*, contrasting maximum profit and maximum welfare solutions and assuming demand in terms of passenger miles to be dependent on price and bus miles operated. Panzar [1979] analyses *infrequent airline services*, assuming demand to be dependent on price and service frequency and allowing for a distribution of ideal departure times. These works consider demand from all passengers, or from one representative group travelling the average distance, with no concern for where passengers board and alight. Jansson [1991] considers and contrasts *frequent and infrequent services*, and takes into account *a variety of passenger groups*. This work follows Jansson in the basic modelling approach, but deals also with (a) two passenger groups with different valuations, (b) that the railway operators may either be welfare or profit maximising, and (c) competition between rail operators or between a rail operator and other modes.

We will first, in section 2, define the basic prerequisites and assumptions, including the definition of the passengers' price and quality attributes. Section 3 deals with welfare and profit maximisation for one service and one passenger group. Section 4 extends the analysis to deal with several passenger groups. Section 5 analyses the implications of external effects and second-best situations. Section 6 summarises the main results of parallel work on simulations of subsidies applied on the Swedish rail network. The main results are summarised in section 7.

## 2 BASIC PREREQUISITES AND ASSUMPTIONS

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## 2.1 Notation

- $\sigma$ is the number of seats per carriage.
- I is the variable infrastructure cost per departure due to track maintenance etc.,
- F is frequency in number of departures per hour,
- f is the corresponding infrastructure fee per departure,
- e is the external cost per departure,
- $r_i$ is the ride time of group i,
- $c_{Li}$ is a cost proportionate to number of passengers in group i, mainly sales costs,
- $\varphi$ is the vector of the travel time components,
- $\sigma_1$ and $\sigma_1$ are the number of seats in per carriage 1st and 2nd class respectively,
- $\phi_i$ is the monetary time value of group i,
- $\phi^\tau$ is the monetary time value of frequency delay (wait time) of group i,
- $\tau_i$ is frequency delay of group i,
- $T_i^T$ is the cost of frequency delay of group i
- $N_1$ and $N_2$ are the numbers of first and second class carriages used in a train,
- h is the round trip distance of a line,
- $c_{\sigma 1}$ and $c_{\sigma 2}$ are the per unit capital and personnel costs for first and second class carriages,
- $C_F$ is costs related to the each departure of the train (but not related to the train size), i.e., capital costs of the minimum train size, including cost of the driver, terminal costs, energy costs etc,
- $C_N[N_{1k}+N_{2k}]$ are the costs directly related to the train size, i.e., certain personnel costs (e.g. conductors, cleaners), energy, the depot size, the platform size etc. The derivative of this cost with respect to size is assumed to increase strongly over a certain size,
- X1 and X2 is the number of passengers in each group 1 and 2 during a period of time, thought of as one hour.

We introduce the following notation:

$\varepsilon \equiv \frac{\partial X}{\partial p}\frac{p}{X}$ for own price elasticity,

$\varepsilon_F \equiv \frac{\partial X}{\partial F}\frac{F}{X}$ for frequency elasticity,

$\varepsilon_N \equiv \frac{\partial X}{\partial N}\frac{N}{X}$ for elasticity with respect to number of carriages (here also denoted carriage elasticity).

Differentials are written in two different ways, either by use of deltas or by use of subindex, e.g.,

$$\frac{\partial X}{\partial F} \equiv X_F$$

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A specific service is denoted k, which could be a railway line, a coach line or an airline.

Arguments of functions are throughout delimited by [], while polynoms are delimited by ().

## 2.2 Authorities

All the infrastructure authorities, for road, rail and air, are assumed to be welfare maximising. However, we will also investigate the consequences of not applying optimal infrastructure charges by a second-best analysis.

The Railway Track Authority (RTA) is assumed to be responsible for investments and maintenance of tracks, electricity distribution and allocation of slots (time spaces between departures) for rail operation. The RTA is also charging a track user charge.

## 2.3 The operating firms

The firms may be either welfare or profit maximising. In both cases they optimise prices, service frequency and the number of carriages of each kind for a certain period of time during the day.

Generally we will assume efficiency in production, i.e., that any level of output is produced at minimum cost, irrespective of whether the actual producer is welfare or profit maximising. The focus is put on consumption efficiency related to determination of optimal prices, optimal frequency, optimal number of carriages and possible subsidies and infrastructure charges.

Demand is assumed to be specified for certain periods, such as the average weekday afternoon peak hour in wintertime, the average Saturday etc. Only one type of charge - a per-trip price - is considered.

The operating firm reaches decisions about relevant inputs and prices well ahead of implementation because of a necessary planning lag. All factors of production that are variable between decision and implementation are therefore considered relevant for the joint decision on the magnitude of policy variables. These factors include, we assume, the number and size of trains, as well as personnel required at various levels of demand and by various numbers of units, and give rise to what we call variable costs. Policy variables are thus considered to be continuous. This assumption is not very restrictive, neither with respect to frequency, nor with respect to number of carriages. A theoretical optimal number of carriages at 6.7 could in practice be 6 or 7 etc.

If C[N] denotes the cost per departure, total operating costs per hour for a line is:

$$(1)\, FC[N] + c_L X \equiv F(C_F + C_N[N])h + c_L X$$

The notation for distance h is for simplicity reasons subsequently omitted, bearing in mind that all departure related costs should be multiplied with the factor h. Infrastructure costs of rail operation is IF. Finally production gives rise to external costs eF. The operators are supposed to pay the infrastructure charge fF. It could be, or not, that f=I+e. There are also

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basic administrative and planning costs, A. These costs A are exogenous in the model since they are not affected by the operation of a specific line.

## 2.4 The passengers

Since first and second class passengers use different carriages, passengers in each group affect only fellow passengers belonging to the same group.

Aggregate consumers' surplus is expressed as a function of "generalised cost", $G = p + \phi\varphi$, where $\varphi$ is the vector of the travel time components and $\phi$ is the vector of monetary time values, i.e., the marginal rates of substitution between price and travel time components. Although $\phi$ is assumed to be the same for all individuals within a group, i.e., the same for all at each point $[p, \varphi]$, $\phi$ may be a function of $\varphi$ and vary among passenger groups. For the sake of simplicity, however, $\phi$ is here often written without index for group. The vector $\varphi$ is here assumed to comprise riding time and "frequency delay", which is the time interval between ideal and actual departure time.

Riding time for group i is written $r_i$. The cost of riding time for a group i is then:

$$(2) T_i \equiv \phi_i[\frac{X_i}{FN_i\sigma_i}]r_i$$

where

$$(2b) \frac{\partial T_i}{\partial X_i} = \frac{\partial \phi_i}{\partial D_i}\frac{1}{FN_i\sigma_i}r_i$$

$\phi_i$ is thus the value of riding time, assumed to be dependent on the occupancy rate, $D_i \equiv X_i/FN_i\,\sigma_i$, which is the number of passengers per seat in a group, and where it is assumed that $\partial\phi/\partial D > 0$ and $\partial^2\phi/\partial D^2 > 0$, so that the value of riding time is progressive with the occupancy rate. The number of passengers per departure is $q_i \equiv X_i/F$. We ignore that different passenger groups travel different distances, $r_i$. We interpret r instead as the average distance travelled. Subsequently we omit the ride time r, bearing in mind that ride time costs should be multiplied by the time r. These simplifications will substantially facilitate the expressions derived without disturbing the purpose of this analysis. The consequences of taking into account various distances travelled will be commented where appropriate and the full analysis of this aspect is found in Jansson [1991].

The interval between departures is 1/F hours. Ideal departure times, t, are uniformly distributed within this interval, i.e., $0 < t \leq 1/F$. Frequency delay is then $\tau \equiv 1/F - t$. The cost of frequency delay for a group is $T^{\tau}[\phi^{\tau}, F, t] = \phi^{\tau}[1/f-t](1/F-t)$. This is the delay multiplied by value of time, which is a function of the delay. The value of time of 1st class passengers could be assumed to be substantially higher than the value of time of 2nd class passengers. Subsequently we ignore the distribution of t and assume that the expectation value of the frequency delay is a function of frequency only, i.e $T^{\tau} = T^{\tau}[F]$. Ignoring the distribution t drastically simplifies the analysis without significant drawbacks for the purpose of this paper, but the full effects of taking t into account is found in Jansson [1991]. It may be useful to

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$$(3) G[p,F,N,s] \equiv p + T + T^{\tau}[F] \equiv p + \phi\left[\frac{X}{FN\sigma}\right] + \frac{\phi_F}{2F}$$

think about the frequency delay as wait time calculated as half the headway, 1/2F, and wait time cost as $\phi^{\tau}$ /2F.

If p denotes the price, the generalised cost of travel for a group (index omitted) at time t is:

$$(3) G[p,F,N,s] \equiv p + T + T^{\tau}[F] \equiv p + \phi\left[\frac{X}{FN\sigma}\right] + \frac{\phi^{\tau}}{2F}$$

so that:

$$(3b) \frac{\partial G}{\partial p} = 1 + \frac{\partial T}{\partial X}\frac{\partial X}{\partial p}$$

Demand per hour, X, for each group is a function of generalised cost:

$$(4) X = X[G[p,\ F,\ N,\sigma]]$$

Note that the number of passengers in the same group affects demand in a certain group but not in other groups, since the groups use different carriages. We know that own-price elasticities, denoted $\varepsilon_p$, are negative, $\varepsilon_p<0$. We assume, based on solid empirical evidence, that demand elasticities with respect to frequency, vehicle size and train size, denoted $\varepsilon_F$, $\varepsilon_N$ and $\varepsilon_\sigma$, are such that $0<\varepsilon_F<1$, $0<\varepsilon_N<1$, implying that $\partial q/\partial F<0$, $\partial q/\partial N<0$. This rules out the possibility that an increase in frequency or unit size would generate so many passengers that the occupancy rate is unchanged or increases[1]. Aggregate consumers' surplus for a group is:

$$(5) S[G] = \int_{G}^{G^{max}} X[\rho]d\rho$$

and

$$(5b) \frac{\partial S}{\partial G} = -X$$

$G^{max}$ is here the reservation price in generalised cost terms for the individual in a group i with the maximum reservation price in generalised cost terms.

We should already here note the following important relationships with respect to frequency and number of carriages.

$$(6) \frac{\partial S}{\partial G}\frac{\partial G}{\partial F} \equiv -X\frac{\partial T^{\tau}}{\partial F} - X\left(\frac{\partial G}{\partial T}\frac{\partial T}{\partial X}\frac{\partial X}{\partial F} + \frac{\partial G}{\partial T}\frac{\delta T}{\delta F}\right) \equiv -X\frac{\partial T^{\tau}}{\partial F} - X\left(\frac{\partial T}{\partial X}\frac{\partial X}{\partial F} + \frac{\delta T}{\delta F}\right)$$

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[1] Taking elasticity with respect to frequency as an example, we have:

$$\frac{\partial q}{\partial F} \equiv \frac{F\frac{\partial X}{\partial F} - X}{F^2} \equiv \frac{X\left(\frac{F}{X}\frac{\partial X}{\partial F} - 1\right)}{F^2} \equiv \frac{X(\varepsilon_F - 1)}{F^2}$$ which is<0 only if $\varepsilon_F<1$

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$$(7)\frac{\partial S}{\partial G}\frac{\partial G}{\partial N} \equiv -X(\frac{\partial G}{\partial T}\frac{\partial T}{\partial X}\frac{\partial X}{\partial N} + \frac{\partial G}{\partial T}\frac{\delta T}{\delta N}) \equiv -X(\frac{\partial T}{\partial X}\frac{\partial X}{\partial N} + \frac{\delta T}{\delta N})$$

Expression (6) shows the consumer surplus change of a marginal change of frequency. The first term on the right hand side is the effect on frequency delay. This term can also be said to represent a positive external effect of public transport due to economies of scale in consumption. The second term reflects the effect on ride time cost. This cost is composed of two terms. The first one is the effect via demand, which in turn affects the crowding in the train. The second effect is the direct effect on crowding of a frequency change, denoted by use of the special delta $\delta$. By differentiation of (2) with respect to F we can also write these two effects as follows.

$$(8)X\frac{\partial \phi}{\partial D}(\frac{FX_F}{F^2 N\sigma} - \frac{X}{F^2 N\sigma}) \equiv X\frac{\partial T}{\partial X}(\frac{\partial X}{\partial F} - \frac{X}{F})$$

Expression (7) shows the consumer surplus change of a marginal change of number of carriages. This cost is composed of two terms. The first one is the effect via demand, which in turn affects the crowding in the train. The second effect is the direct effect on crowding of a change of number of carriages, denoted by use of the special delta $\delta$. By differentiation of (2) with respect to N we can also write these two effects as follows.

$$(9)X\frac{\partial \phi}{\partial D}(\frac{NX_N}{FN^2\sigma} - \frac{X}{FN^2\sigma}) \equiv X\frac{\partial T}{\partial X}(\frac{\partial X}{\partial N} - \frac{X}{N})$$

Below we derive the basic marginal effects on demand with respect to price and frequency.

$$(10)\frac{\partial X}{\partial p} = \frac{\partial X}{\partial G}\frac{\partial G}{\partial p} \equiv \frac{\partial X}{\partial G}(\frac{\delta G}{\delta p} + \frac{\partial G}{\partial T}\frac{\partial T}{\partial X}\frac{\partial X}{\partial p})$$

$$(11)\frac{\partial X}{\partial p}(1 - \frac{\partial X}{\partial G}\frac{\partial T}{\partial X}) = \frac{\partial X}{\partial G}$$

$$(12)\frac{\partial X}{\partial p} = \frac{\frac{\partial X}{\partial G}}{(1 - \frac{\partial X}{\partial G}\frac{\partial T}{\partial X})}$$

$$(13)\frac{\partial G}{\partial F} = (\frac{\delta T}{\delta F} + \frac{\partial T}{\partial X}\frac{\partial X}{\partial F}) + \frac{\partial T^{\tau}}{\partial F} \equiv \frac{\partial T}{\partial X}(-\frac{X}{F} + \frac{\partial X}{\partial F}) + \frac{\partial T^{\tau}}{\partial F}$$

$$(14)\frac{\partial X}{\partial F} = \frac{\partial X}{\partial G}\frac{\partial G}{\partial F} \equiv \frac{\partial X}{\partial G}\frac{\partial G}{\partial T}(\frac{\delta T}{\delta F} + \frac{\partial T}{\partial X}\frac{\partial X}{\partial F}) + \frac{\partial X}{\partial G}\frac{\partial G}{\partial T^{\tau}}\frac{\partial T^{\tau}}{\partial F}$$

$$(15)\frac{\partial X}{\partial F}(1 - \frac{\partial X}{\partial G}\frac{\partial T}{\partial X}) \equiv \frac{\partial X}{\partial G}(\frac{\delta T}{\delta F} + \frac{\partial T^{\tau}}{\partial F}) \equiv \frac{\partial X}{\partial G}(-\frac{\partial T}{\partial X}\frac{X}{F} + \frac{\partial T^{\tau}}{\partial F})$$

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$$(16)\ \frac{\partial X}{\partial F}=\frac{\frac{\partial X}{\partial G}\left(\frac{\delta T}{\delta F}+\frac{\partial T^{\tau}}{\partial F}\right)}{\left(1-\frac{\partial G}{\partial T}\frac{\partial T}{\partial X}\right)}\equiv\frac{\frac{\partial X}{\partial G}\left(-\frac{\partial T}{\partial X}\frac{X}{F}+\frac{\partial T^{\tau}}{\partial F}\right)}{\left(1-\frac{\partial G}{\partial T}\frac{\partial T}{\partial X}\right)}$$

$$(17)\ \frac{\partial X}{\partial G}=\frac{\frac{\partial X}{\partial F}}{T_X\left(\frac{\partial X}{\partial F}-\frac{X}{F}\right)+\frac{\partial T^{\tau}}{\partial F}}$$

$$(18)\ \frac{X}{X_G}=\frac{XT_X\left(\frac{\partial X}{\partial F}-\frac{X}{F}\right)+X\frac{\partial T^{\tau}}{\partial F}}{X_F}\equiv XT_X-XT_X\frac{\frac{X}{F}}{X_F}+\frac{X\frac{\partial T^{\tau}}{\partial F}}{X_F}$$

$$(19)\ \frac{\partial G}{\partial N}=\left(\frac{\delta T}{\delta N}+\frac{\partial T}{\partial X}\frac{\partial X}{\partial N}\right)\equiv\frac{\partial T}{\partial X}\left(\frac{\partial X}{\partial N}-\frac{X}{N}\right)$$

$$(20)\ \frac{\partial X}{\partial N}=\frac{\partial X}{\partial G}\frac{\partial G}{\partial N}\equiv\frac{\partial X}{\partial G}\frac{\partial G}{\partial T}\left(\frac{\delta T}{\delta N}+\frac{\partial T}{\partial X}\frac{\partial X}{\partial N}\right)$$

$$(21)\ \frac{\partial X}{\partial N}\left(1-\frac{\partial X}{\partial G}\frac{\partial T}{\partial X}\right)\equiv\frac{\partial X}{\partial G}\frac{\delta T}{\delta N}\equiv-\frac{\partial X}{\partial G}\frac{\partial T}{\partial X}\frac{X}{N}$$

$$(22)\ \frac{\partial X}{\partial N}=\frac{\frac{\partial X}{\partial G}\frac{\delta T}{\delta N}}{\left(1-\frac{\partial G}{\partial T}\frac{\partial T}{\partial X}\right)}\equiv\frac{-\frac{\partial X}{\partial G}\frac{\partial T}{\partial X}\frac{X}{N}}{\left(1-\frac{\partial G}{\partial T}\frac{\partial T}{\partial X}\right)}$$

$$(23)\ \frac{\partial X}{\partial G}=\frac{\frac{\partial X}{\partial N}}{\frac{\partial T}{\partial X}\left(\frac{\partial X}{\partial N}-\frac{X}{N}\right)}$$

$$(24)\ \frac{X}{X_G}=\frac{XT_X\left(\frac{\partial X}{\partial N}-\frac{X}{N}\right)}{X_N}\equiv XT_X-XT_X\frac{\frac{X}{N}}{X_N}$$

# 3 ONE SERVICE AND ONE PASSENGER GROUP

## 3.1 Objective functions

We present here the objective functions of the welfare maximisation and the profit maximisation models. In this section we do not consider several services and several passenger groups for which reason indices are omitted. We neither consider a binding budget constraint.

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The maximisation relates to one service during a period normalised to one (hour). The analysis may then be repeated for other periods and routes.

The welfare objective function is expressed as:

$$(25) W = S[G[p, F, N]] + pX[p, F, N] - FC[N] - c_L X - IF - eF$$

The objective function for profit maximisation includes only producer's surplus, taking into account the infrastructure paid, f:

$$(26) \pi = pX[p, F, N] - FC[N] - c_L X - fF$$

## 3.2 Welfare optimum

**Prices**

The first-order conditions with respect to the price yields:

$$(27) \frac{\partial W}{\partial p} = \frac{\partial S}{\partial G}\frac{\partial G}{\partial p} + X + p\frac{\partial X}{\partial p} - c_L\frac{\partial X}{\partial p} = 0$$

where

$$\frac{\partial S}{\partial G} = -X$$

and

$$\frac{\partial G}{\partial p} = 1 + \frac{\partial T}{\partial X}\frac{\partial X}{\partial p}$$

The optimal price is then:

$$(28) p^{*W} = c_L + XT_X$$

The welfare optimal price is thus composed of the marginal production cost plus the marginal passenger cost with respect to number of passengers.

By use of (2b) we can also express the optimal price as:

$$(28b) p^{*W} = c_L + \frac{\partial \phi}{\partial D}\frac{X}{FN\sigma}$$

We also achieve an expression for optimal frequency:

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$$(28c) F^{*W} = \frac{X\frac{\partial \phi}{\partial D}}{N\sigma(p^{*W} - c_L)}$$

**Frequency**

The first-order condition with respect to frequency yields:

$$(29) \frac{\partial W}{\partial F} = -X\frac{\partial G}{\partial F} + p\frac{\partial X}{\partial F} - c_L\frac{\partial X}{\partial F} - C - I - e = 0$$

$$(30) \frac{\partial W}{\partial F} = -\frac{X}{X_G} + p - c_L - \frac{C + I + e}{X_F} = 0$$

$$(31) p^{*W} = c_L + \frac{X}{X_G} + \frac{C + I + e}{X_F}$$

We use (18) in order to achieve an alternative expression for the optimal price.

$$(32) p^{*W} = c_L + XT_X(\frac{X_F - X/F}{X_F}) + \frac{X\frac{\partial T^{\tau}}{\partial F}}{X_F} + \frac{C + I + e}{X_F}$$

$$(33) p^{*W} = c_L + \frac{1}{\varepsilon_F}(\frac{F(C + I + e)}{X} + FT_X(X_F - X/F) + F\frac{\partial T^{\tau}}{\partial F})$$

The first term within the parenthesis in (33) is the average variable cost besides the sales cost. The second term is negative since $X/F > X_F$. $X/F$ stems from the direct effect on in-vehicle crowding of a frequency change and $X_F$ stems from the indirect effect on in-vehicle crowding, via demand change, of a frequency change. The optimal price is thus reduced below the average variable cost due to the net positive effect on crowding of a frequency increase. The third term represents the positive effect on wait time cost of a frequency change and will also push the optimal price downwards below the average variable cost. All costs apart from the marginal sales cost are increased by the inverse of the frequency elasticity.

**Carriages**

The first-order condition with respect to number of carriages yields:

$$(34) \frac{\partial W}{\partial N} = -X\frac{\partial G}{\partial N} + p\frac{\partial X}{\partial N} - c_L\frac{\partial X}{\partial N} - F\frac{\partial C_N}{\partial N} = 0$$

$$(35) \frac{\partial W}{\partial N} = -\frac{X}{X_G} + p - c_L - \frac{F\frac{\partial C_N}{\partial N}}{X_N} = 0$$

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$$(36)\, p^{*W} = c_L + \frac{X}{X_G} + \frac{F\frac{\partial C_N}{\partial N}}{X_N}$$

$$(37)\, p^{*W} = c_L + \frac{F\frac{\partial C_N}{\partial N}}{X_N} + \frac{XT_X(\frac{\partial X}{\partial N} - \frac{X}{N})}{X_N}$$

$$(38)\, p^{*W} = c_L + \frac{1}{\varepsilon_N}\left(\frac{NF}{X}\frac{\partial C_N}{\partial N} + NT_X\left(\frac{\partial X}{\partial N} - \frac{X}{N}\right)\right)$$

The first term within the parenthesis is the marginal cost of a changed number of carriages per passenger. The second term is negative since $X/N > X_N$. $X/N$ stems from the direct effect on in-vehicle crowding of a changed number of carriages and $X_N$ stems from the indirect effect on in-vehicle crowding, via demand change, of a changed number of carriages. The optimal price is thus reduced below the average variable cost due to the net positive effect of a changed number of carriages.

**Combination of welfare optima for price and frequency**

Equality between optimal price according to the first-order conditions with respect to price (28) and frequency (31) yields:

$$(39)\, XT_X - \frac{X}{X_G} = \frac{C + I + e}{X_F}$$

Use of (18) yields:

$$(40)\, \frac{XT_X\frac{X}{F} - X\frac{\partial T^{\tau}}{\partial F}}{X_F} = \frac{C + I + e}{X_F}$$

$$(41)\, F^{*W} = \frac{X^2 T_X}{C + I + e + X\frac{\partial T^{\tau}}{\partial F}}$$

The numerator shows that the optimal frequency grows with in-vehicle congestion and demand. The denominator shows that the optimal frequency declines with the costs but grows with the effect on frequency delay of a marginal frequency increase.

We can then also express optimal price in an alternative way by noting that:

$$(42)\, \frac{F(C + I + e + X\frac{\partial T^{\tau}}{\partial F})}{X} = XT_X \equiv p - c_L$$

We achieve:

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$$(43) p^{*W} = c_L + \frac{F(C + I + e)}{X} + F\frac{\partial T^{\tau}}{\partial F}$$

The welfare optimal price is thus equal to the marginal production cost (sales cost) plus the average variable operating, and external, cost plus the marginal effect of a frequency change. This effect on reduced frequency delay cost was also seen in (33) above. Note that when the optimal price is expressed in this way one cannot see the positive effect related to in-vehicle crowding that was visible in (33).

**Combination of welfare optima for price and carriages**

Equality between optimal price for the first-order conditions with respect to price and number of carriages yields:

$$(44) XT_X - \frac{X}{X_G} = \frac{F\frac{\partial C_N}{\partial N}}{X_N}$$

$$(45) XT_X = XT_X - XT_X\frac{\frac{X}{N}}{X_N} + \frac{F\frac{\partial C_N}{\partial N}}{X_N}$$

$$(46) XT_X\frac{X}{N} = F\frac{\partial C_N}{\partial N}$$

$$(47) N^{*W} = \frac{X^2 T_X}{F\frac{\partial C_N}{\partial N}}$$

By combining (47) with (28) we get:

$$(47b) p^{*W} = c_L + \frac{NF\frac{\partial C_N}{\partial N}}{X}$$

The optimal price is thus the marginal production cost plus the marginal cost with respect to number of carriages, per passenger.

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**Combination of welfare optima for price, frequency and carriages**

By combining (43) and (47b) we get: get:

$$(47c)\frac{F(C+I+e)}{X}+F\frac{\partial T^{\tau}}{\partial F}\equiv\frac{F(C+I+e)}{X}-\frac{\phi^{\tau}}{2F}=\frac{NF\frac{\partial C_N}{\partial N}}{X}$$

$$(47d)F^2=\frac{X\phi^{\tau}}{2(C+I+e-N\frac{\partial C_N}{\partial N})}\equiv\frac{X\phi^{\tau}}{2(C_F+C_N[N]+I+e-N\frac{\partial C_N}{\partial N})}$$

Again we note that optimal frequency grows with the value of frequency delay. The new information provided by (47d) is that if the cost with respect to number of carriages is proportionate to number of carriages then the denominator would only include operation cost, infrastructure cost and external cost. If on the other hand the carriage costs are progressive with number of carriages, the denominator would be reduced below the other costs and the supply would rather increase by optimal frequency rather than the growing carriage costs.

## 3.3 Profit optimum

**Prices**

The first-order condition with respect to price yields:

$$(48)\frac{\partial\pi}{\partial p}=X+pX_p-c_LX_p=0$$

$$(49)p^{*\pi}=c_L-\frac{X}{\partial X/\partial p}$$

By use of the price elasticity concept, optimal price can also be written as:

$$(50)p^{*\pi}=\frac{c_L}{1+\frac{1}{\varepsilon_p}}$$

By using the development of the differential of X with respect to p in (12) we can also write (49) as:

$$(51)p^{*\pi}=c_L-\frac{X}{X_p}\equiv c_L-\frac{X(1-X_GT_X)}{X_G}\equiv c_L-\frac{X}{X_G}+XT_X$$

By use of (18) we can also write (51) as:

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$$(51b) p^{*\pi} = c_L + XT_X - \frac{X\,T_X(\varepsilon_F - 1)}{\varepsilon_F} - \frac{F\frac{\partial T^{\tau}}{\partial F}}{\varepsilon_F}$$

**Frequency**

The first-order condition with respect to frequency yields:

$$(52) \frac{\partial \pi}{\partial F} = p\frac{\partial X}{\partial F} - c_L\frac{\partial X}{\partial F} - C - f = 0$$

$$(53) p^{*\pi} = c_L + \frac{C + f}{X_F} \equiv c_L + \frac{F(C + f)}{X\varepsilon_F}$$

**Carriages**

The first-order condition with respect to number of carriages yields:

$$(54) \frac{\partial \pi}{\partial N} = p\frac{\partial X}{\partial N} - c_L\frac{\partial X}{\partial N} - F\frac{\partial C_N}{\partial N} = 0$$

$$(55) p^{*\pi} = c_L + \frac{F\frac{\partial C_N}{\partial N}}{X_N} \equiv c_L + \frac{NF\frac{\partial C_N}{\partial N}}{X\varepsilon_N}$$

**Combination of profit optima for price and frequency**

Again, equality between optimal price for the first-order conditions with respect to price and frequency yields:

$$(56) XT_X - \frac{X}{X_G} = \frac{C + f}{X_F}$$

Use of (18) yields:

$$(57) \frac{XT_X\frac{X}{F} - X\frac{\partial T^{\tau}}{\partial F}}{X_F} = \frac{C + f}{X_F}$$

$$(58) F^{*\pi} = \frac{X^2T_X}{C + f + X\frac{\partial T^{\tau}}{\partial F}}$$

We can then also express optimal price in an alternative way by noting that:

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$$(59)\frac{F(C+f+X\frac{\partial T^{\tau}}{\partial F})}{X}=XT_X\equiv p-c_L+\frac{X}{X_G}$$

We achieve:

$$(60)p^{*\pi}=c_L+\frac{F(C+f)}{X}+F\frac{\partial T^{\tau}}{\partial F}-\frac{X}{X_G}$$

$$(61)p^{*\pi}=c_L+\frac{F(C+f)}{X}+F\frac{\partial T^{\tau}}{\partial F}-\frac{X\frac{\partial T}{\partial X}(\frac{\partial X}{\partial F}-\frac{X}{F})+X\frac{\partial T^{\tau}}{\partial F}}{X_F}\equiv$$

$$\equiv c_L+\frac{F(C+f)}{X}-\frac{X\frac{\partial T}{\partial X}(\varepsilon_F-1)}{\varepsilon_F}+\frac{F\frac{\partial T^{\tau}}{\partial F}(\varepsilon_F-1)}{\varepsilon_F}$$

According to (60) the profit optimal price exceeds the welfare optimal price with $-X/X_G$, given that f=I+e. Expression (61) demonstrates the difference in another way. The indirect of a frequency change, via demand, pushes the price upwards and above the average cost. The direct effect of a frequency change, however, pushes the price downwards and the latter effect is larger. The net effect is a price exceeding the welfare optimal price but also exceeding the average cost.

This net effect that stems from the deviation of frequency elasticity from zero is thus the margin for coverage of fixed costs and pure profit.

**Combination of profit optima for price and carriages**

Again, equality between optimal price for the first-order conditions with respect to price and number of carriages yields:

$$(62)XT_X-\frac{X}{X_G}=\frac{F\frac{\partial C_N}{\partial N}}{X_N}$$

Consequently:

$$(63)N^{*\pi}=\frac{X^2T_X}{F\frac{\partial C_N}{\partial N}}$$

The expression for optimal number of carriages under profit maximisation thus looks the same as the corresponding expression under welfare maximisation.

We can also express the optimal price in an alternative way by noting that:

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$$(63b)\frac{NF\frac{\partial C_N}{\partial N}}{X} = XT_X \equiv p - c_L + \frac{X}{X_G}$$

We achieve:

$$(63c)p^{*\pi} = c_L + \frac{NF\frac{\partial C_N}{\partial N}}{X} - \frac{X\frac{\partial T}{\partial X}(\frac{\partial X}{\partial N} - \frac{X}{N})}{X_N} \equiv c_L + \frac{NF\frac{\partial C_N}{\partial N}}{X} - \frac{X\frac{\partial T}{\partial X}(\varepsilon_N - 1)}{\varepsilon_N}$$

**Diagnostic comparison between welfare and profit optima**

Below we repeat and compare the various expressions for optimal price according to welfare and profit maximisation. In these comparisons we assume that the infrastructure charge paid by profit maximising operators equals the infrastructure costs and external costs, so that f=I+e.

$$(28)p^{*W} = c_L + XT_X$$

$$(51b)p^{*\pi} = c_L + XT_X - \frac{X\,T_X(\varepsilon_F - 1)}{\varepsilon_F} - \frac{F\frac{\partial T^\tau}{\partial F}}{\varepsilon_F}$$

Here we note that the profit optimal price firstly exceeds the welfare optimal price by the difference between the direct and the indirect effects on riding time cost. Secondly the profit maximiser adds the passengers' marginal frequency delay cost. Both additions are multiplied with the inverse of the frequency elasticity.

$$(43)p^{*W} = c_L + \frac{F(C + I + e)}{X} + F\frac{\partial T^\tau}{\partial F}$$

$$(53)p^{*\pi} = c_L + \frac{F(C + f)}{X\varepsilon_F}$$

$$(61)p^{*\pi} = c_L + \frac{F(C + f)}{X} - \frac{X\frac{\partial T}{\partial X}(\varepsilon_F - 1)}{\varepsilon_F} + \frac{F\frac{\partial T^\tau}{\partial F}(\varepsilon_F - 1)}{\varepsilon_F}$$

(43) demonstrates that the welfare optimal price is equal to the average variable operating cost minus a term that reflects the positive effect of a frequency increase.

(53) shows that the profit maximiser ignores the positive effect of a frequency increase. In addition the average production and external costs are increased by the inverse of the frequency elasticity.

(61) shows another way to express the difference. Now no longer the production and external costs are upgraded. Instead the following two terms show that the positive effect of a frequency increase is counterweighted since $\varepsilon_F < 1$. It is the direct effects on in-vehicle

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crowding and frequency delay that are ignored and these effects are larger than the indirect effect via demand change that the profit maximiser takes into account.

In total, while the welfare optimal price is below the average variable cost, the profit maximising price is above the average variable cost.

$$(47b)\; p^{*W} = c_L + \frac{NF\frac{\partial C_N}{\partial N}}{X}$$

$$(55)\; p^{*\pi} = c_L + \frac{NF\frac{\partial C_N}{\partial N}}{X\varepsilon_N}$$

$$(63c)\; p^{*\pi} = c_L + \frac{NF\frac{\partial C_N}{\partial N}}{X} - \frac{X\frac{\partial T}{\partial X}(\varepsilon_N - 1)}{\varepsilon_N}$$

(55) shows that the profit maximiser upgrades the average marginal cost with respect to number of carriages with the inverse of the elasticity with respect to number of carriages.

(63c) shows another way to express the difference. Now no longer the average marginal cost with respect to number of carriages is upgraded. Instead the following term shows that the direct effect on in-vehicle crowding is ignored and this effects is larger than the indirect effect via demand change that the profit maximiser takes into account.

With respect to optimal frequency we know that the expressions, repeated below, are the same for welfare and profit maximisation, given that the infrastructure charge equals the infrastructure operating cost and the external operating cost, i.e., that f=I+e.

$$(41)\; F^{*W} = \frac{X^2 T_X}{C + I + e + X\frac{\partial T^{\tau}}{\partial F}}$$

$$(58)\; F^{*\pi} = \frac{X^2 T_X}{C + f + X\frac{\partial T^{\tau}}{\partial F}}$$

Also with respect to optimal number of carriages we know that the expressions, repeated below, are the same for welfare and profit maximisation:

$$(47)\; N^{*W} = \frac{X^2 T_X}{F\frac{\partial C_N}{\partial N}}$$

$$(63)\; N^{*\pi} = \frac{X^2 T_X}{F\frac{\partial C_N}{\partial N}}$$

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**Comparative sizes of the optima for welfare and profit maximisation**

Let us now evaluate the relationships between optimal price, frequency and number of carriages for welfare and profit maximisation respectively.

The point of departure is the derived fact that if optimal frequency and number of carriages were the same for the two objectives, then all the expressions for optimal prices show that the optimal price is higher for profit than for welfare maximisation. We will then use the expressions for optimal frequency, (41) and (58), and number of carriages, (47) and (63) in order to investigate the following. Is the optimal frequency larger or smaller for profit maximisation given that the optimal number of carriages is the same and is the optimal number of carriages larger or smaller for profit maximisation given that the optimal frequency is the same?

*Is the optimal frequency higher or lower for profit or welfare maximisation?*

Assume that the number of carriages is the same for both objective functions.

The higher price with profit maximisation means that demand X and the marginal crowding cost $T_X$ are lower for profit than for welfare maximisation, so the numerator is lower for profit than for welfare maximisation. Since X is lower for profit maximisation and the marginal cost with respect to frequency delay is the same if frequency is the same we also know that the denominator is higher for profit than for welfare maximisation.

The conclusion is that expression (58) for optimal frequency cannot be fulfilled if frequencies are the same, but only if optimal frequency is lower for profit than for welfare maximisation.

*Is the optimal number of carriages higher or lower for profit or welfare maximisation?*

Assume that the frequency is the same for both objective functions.

The higher price with profit maximisation means that demand X and the marginal crowding cost $T_X$ are lower for profit than for welfare maximisation, so the numerator is lower for profit than for welfare maximisation. Since the marginal cost with respect to number of carriages is the same if the number of carriages is the same we also know that the denominator is higher for profit than for welfare maximisation.

The conclusion is that expression (58) for optimal number of carriages cannot be fulfilled if the number of carriages is the same, but only if the number of carriages is lower for profit than for welfare maximisation.

In total we can conclude that:

$P^{*\pi} > P^{*W}$ and $F^{*\pi} < F^{*W}$ at least if $N^{*\pi} = N^{*W}$

$P^{*\pi} > P^{*W}$ and $N^{*\pi} < N^{*W}$ at least if $F^{*\pi} = F^{*W}$

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## 3.5 Corrections for non-optimal behaviour

In order for the welfare maximising authority to deal with the non-optimal behaviours of the profit maximising competitors two policy parameters are introduced. One is a possible subsidy, motivated by the result in section 4 where subsidies where found to make the operator act in the direction towards social optimum. The other parameter is an infrastructure charge, or producer fee, meant to complement the subsidy in order to be able to care for both the price and the frequency variables. We ignore a possible third parameter that could be a consumption tax, VAT for example. Introduction of such a tax would, however, mean no difference to the principle outcome of the analysis since it would only serve as a fiscal parameter that would affect the magnitudes of the subsidy and the producer fee but not their principle influences.

We thus introduce a subsidy, s, per kilometre, which means that the objective function and first-order condition is:

$$(64)\pi = (p+s)X[p,\ F,\ N] - FC[N] - c_L X - fF$$

The first-order condition with respect to price yields:

$$(65)\frac{\partial\pi}{\partial p} = X + (p+s)X_p - C_L X_p = 0$$

$$(66)\,p^{*\pi} = c_L - \frac{X}{\partial X/\partial p} - s$$

$$(67)\,p^{*\pi} = c_L - \frac{X}{X_p} - s \equiv c_L - \frac{X(1 - X_G T_X)}{X_G} \equiv c_L - \frac{X}{X_G} - s + XT_X$$

The first-order condition with respect to frequency yields:

$$(68)\frac{\partial\pi}{\partial F} = (p+s)\frac{\partial X}{\partial F} - C_L\frac{\partial X}{\partial F} - C - f = 0$$

$$(69)\,p^{*\pi} = C_L - s + \frac{C+f}{X_F}$$

Apparently, if s = - $X/X_G$, and if f = I+e, then both first-order conditions with respect to profit maximisation coincides with the first-order conditions with respect to welfare maximisation.

The next issue is whether such equality between profit and welfare maximisation can be achieved without a subsidy, but instead by playing with the infrastructure charge f.

There are thus two policy variables, s and f. We are searching the values of s and f respectively that makes welfare and profit optimum equal. We employ the combined first-order conditions with respect to price and frequency for welfare and profit maximisation.

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For welfare optimum (28) and (39) yields:

$$(70)(p - C_L - XT_X)X_F = C + I + e + \frac{X}{X_G}X_F - XT_X X_F$$

In optimum the right-hand side is equal to zero since the parenthesis on the left-hand side represents price equal to social marginal cost.

For profit optimum (69) yields:

$$(71)(p - C_L)X_F = C + f - sX_F$$

We extend expression (71) in order to make the left-hand side equal to the one for welfare maximisation:

$$(72)(p - C_L - XT_X)X_F = C + f - sX_F - XT_X X_F$$

In order to make expression (72) welfare optimal it is required that the right hand sides of (70) and (72) are equal:

$$(73)C + f - sX_F - XT_X X_F = C + I + e + \frac{X}{X_G}X_F - XT_X X_F$$

$$(74)f - sX_F = I + e + \frac{X}{X_G}X_F$$

There are apparently a number of combinations of s and f that will fulfil the welfare optimum criterion. We can also express the criterion in the following ways:

$$(75)s = \frac{f - I - e}{X_F} - \frac{X}{X_G}$$

$$(76)f = I + e + X_F(s + \frac{X}{X_G})$$

We can exemplify a few combinations of s and f.

If the infrastructure charge equals the infrastructure and external costs, f=I+e, we get that:

$$(77)s = -\frac{X}{X_G}$$

If no infrastructure charge is used we get:

$$(78)s = \frac{-I - e}{X_F} - \frac{X}{X_G}$$

In this case the subsidy is modified in the sense that the passengers are charged for the infrastructure and external costs, divided by the demand change of a marginal frequency

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change. It may very well be the case that the net subsidy is negative. The cost coverage related to infrastructure is thus transferred from the operator to the passengers.

If no subsidy is employed we get that:

$$(79)\, f = I + e + X_F \frac{X}{X_G}$$

In this case the optimal subsidy of the passengers is transferred to subsidisation of the operator.

# 4 ONE SERVICE AND SEVERAL PASSENGER GROUPS

## 4.1 Objective functions

We present here the objective functions of the welfare maximisation and the profit maximisation models. Since first and second class passengers use different carriages, passengers in each group affect only fellow passengers belonging to the same group.

The maximisation relates to one service during a period normalised to one (hour). The analysis may then be repeated for other periods and routes.

$$\begin{aligned}(80)W &= \Sigma_i S_i[G_i[p_i,\ F,\ N_i]]+\\ &+\Sigma_i p_i X_i[p_i,\ F,\ N_i] - FC[N_i] - \Sigma_i c_{Li} X_i - FC - FI - Fe +\\ &+\lambda(\Sigma_i p_i X_i[p_i,\ F,\ N_i] - FC[N_i] - \Sigma_i c_{Li} X_i - FC - FI - \overline{\pi})\end{aligned}$$

; i=1,2

The objective function for profit maximisation includes only producer's surplus, while the consumers' surplus and the budget constraints are missing:

$$(80b)\pi = \Sigma_i p_i X_i[p_i,\ F,\ N_i] - FC[\ N_i\ ] - \Sigma_i c_{Li}[X_i] - fF$$

; i=1,2

In the derivations that follow we omit indices in most situations, keeping in mind that there are two carriage types, each carrying only one passenger group. Since passenger groups do not affect each other and since the expressions for each group, frequency and train size look the same apart from indices, we can be without most of them with pleasure. The only situation in which we need indices is where there is a sum of the two passenger groups on the same line, which means that we use index i for passenger group.

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## 4.2 Welfare optimum

**Prices**

The first-order condition yields:

$$(81)\frac{\partial L}{\partial p} = \frac{\partial S}{\partial G}\frac{\partial G}{\partial p} + (1+\lambda)X \; + (1+\lambda)p\frac{\partial X}{\partial p} - \; (1+\lambda)c_L\frac{\partial X}{\partial p} = 0$$

where

$$\frac{\partial G}{\partial p} \equiv \; 1 \; + \; \frac{\partial T}{\partial X}\frac{\partial X}{\partial p}$$

$$\frac{\partial S}{\partial G} \equiv \; -\, X$$

(81) then yields:

$$(82)\frac{\partial L}{\partial p} = \lambda X + \; (-X\frac{\partial T}{\partial X} + (1+\lambda)p \; - (1+\lambda)c_L)\frac{\partial X}{\partial p} = 0$$

$$(83)p^{*W} = c_L + X\frac{T_X}{1+\lambda} - \frac{\lambda X}{X_p}\frac{1}{1+\lambda}$$

And, by use of the price elasticity:

$$(84)p^{*W} = \frac{c_L + X\frac{T_X}{1+\lambda}}{1+\frac{\lambda}{\varepsilon_p(1+\lambda)}}$$

With a binding budget constraint the consumer cost in terms of in-vehicle congestion is discounted by 1/(1+λ). In this way consumer costs are downscaled in order to make them comparable with the upgraded production costs. Apparently the optimal price declines (grows) with a growing (declining) price elasticity (Ramsey-pricing). Since for a given price level 1st class passengers typically are less price-sensitive than 2nd class passengers are, prices should mainly be raised for 1st class passengers. Price discrimination is thus motivated.

**Frequency**

The first-order condition with respect to frequency is:

$$(85)\frac{\partial L}{\partial F} = \; -\Sigma_i X_i\frac{\partial G_i}{\partial F} + \; (1+\lambda)(\Sigma_i p_i\frac{\partial X_i}{\partial F} - \; C - \; \Sigma_i c_{Li}\frac{\partial X_i}{\partial F} - I) - e$$

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$$(86)\Sigma_i \frac{\partial X_i}{\partial F}(-X_i \frac{\partial T_i}{\partial X_i} + (1+\lambda)p_i - (1+\lambda)c_{Li}) = -\Sigma_i X_i \frac{\partial T_i}{\partial X_i}\frac{X_i}{F} + \Sigma_i X_i \frac{\partial T_i^{\tau}}{\partial F} + (1+\lambda)(C+I) + e$$

In optimum, when prices are optimal, frequency is:

$$(87)F^{*W} = \frac{\Sigma_i X_i^2 \frac{\partial T_i}{\partial X_i}}{(1+\lambda)(C+I) + e + \Sigma_i X_i \frac{\partial T_i^{\tau}}{\partial F}}$$

Assume that 1st class passengers have a high value of wait time and 2nd class passengers have a low value of wait time. The high value will push optimal frequency upward and the low value downwards. 2nd class passengers will thus benefit from 1st class passengers' valuations and 1st class passengers will lose from 2nd class passengers' valuations.

From (82) we know that:

$$(88)X_i \frac{\partial T_i}{\partial X_i} = \frac{\lambda X_i}{\frac{\partial X_i}{\partial p_i}} + (1+\lambda)(p_i - c_{Li})$$

and that

$$(89) - X_i \frac{\partial T_i}{\partial X_i} + (1+\lambda)p_i - (1+\lambda)c_{Li} = \frac{-\lambda X_i}{\frac{\partial X_i}{\partial p_i}}$$

(86) can then be rewritten as:

$$(90)\Sigma_i \frac{\partial X_i}{\partial F}\frac{-\lambda X_i}{\frac{\partial X_i}{\partial p_i}} = -\Sigma_i \frac{X_i}{F}\left(\frac{\lambda X_i}{\frac{\partial X_i}{\partial p_i}} + (1+\lambda)(p_i - c_{Li})\right) + \Sigma_i X_i \frac{\partial T_i^{\tau}}{\partial F} + (1+\lambda)(C+I) + e$$

The average of optimal prices is thus:

$$(91)\frac{\Sigma_i p_i X_i}{\Sigma_i X_i} = \frac{\Sigma_i c_{Li} X_i}{\Sigma_i X_i} + \frac{F(C+I)}{\Sigma_i X_i} + \frac{Fe\frac{1}{1+\lambda}}{\Sigma_i X_i} + \frac{F\Sigma_i \frac{X_i}{\frac{\partial X_i}{\partial p_i}}\left(\frac{\partial X_i}{\partial F} - \frac{X_i}{F}\right)\frac{\lambda}{1+\lambda}}{\Sigma_i X_i} + \frac{F\Sigma_i X_i \frac{\partial T_i^{\tau}}{\partial F}\frac{1}{1+\lambda}}{\Sigma_i X_i}$$

Here we note in the fourth term on the right hand side that the difference between the direct and indirect marginal in-vehicle congestion costs are discounted by $\lambda/(1+\lambda)$. As for the profit maximiser the indirect effect via demand pushes the optimal price downwards while the direct effect pushes the optimal price upwards, and the latter effect is the larger one. As for the first-order condition with respect to price we also note that the fourth term demonstrates that the optimal price declines (grows) with a growing (declining) price elasticity.

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The fifth term that frequency delay costs are discounted by $1/(1+\lambda)$. That is, when a budget constraint is taken into account the fourth and the fifth terms show that the passenger benefits of improved service level are downgraded and the third term shows that the external costs are downgraded.

Assume that 1st class passengers have a high value of wait time and ride time costs and that 2nd class passengers have relatively low values. Assume also that 1st class passengers have low price elasticity and that 2nd class passengers have relatively high price elasticity. (91) then shows that the contribution to cost coverage from 1st class passengers grows with respect to the low price elasticity. This is the ordinary Ramsey-pricing rule. But note also that the contribution from 1st class passengers is reduced with respect to the valuations of ride time and wait time. It is thus an empirical issue whether 1st or 2nd class passengers would contribute most to cost coverage in optimum when no distribution concern is taken.

**Carriages**

The first-order condition with respect to number of carriages is:

$$(92)\frac{\partial L}{\partial N_i} = -X_i\frac{\partial G_i}{\partial N_i} + (1+\lambda)(p_i\frac{\partial X_i}{\partial N_i} - C - c_{Li}\frac{\partial X_i}{\partial N_i} - F\frac{\partial C_{Ni}}{\partial N_i}) = 0$$

Use of (19) yields:

$$(93)\frac{\partial X_i}{\partial N_i}(-X_i\frac{\partial T_i}{\partial X_i} + (1+\lambda)p_i - (1+\lambda)c_{Li}) = -X_i\frac{\partial T_i}{\partial X_i}\frac{X_i}{N_i} + (1+\lambda)(C + F\frac{\partial C_{Ni}}{\partial N_i})$$

In optimum, when prices are optimal, the number of carriages of type i is:

$$(94)N_i^{*W} = \frac{X_i^2\frac{\partial T_i}{\partial X_i}\frac{1}{(1+\lambda)}}{(C + F\frac{\partial C_{Ni}}{\partial N_i})}$$

When the budget constraint is binding the in-vehicle congestion cost is discounted by $1/(1+\lambda)$.

By use of (88) and (89) we get:

$$(95)\frac{\partial X_i}{\partial N_i}\frac{-\lambda X_i}{\frac{\partial X_i}{\partial p_i}} = -\frac{X_i}{N_i}(\frac{\lambda X_i}{\frac{\partial X_i}{\partial p_i}} + (1+\lambda)(p_i - c_{Li})) + (1+\lambda)(C + F\frac{\partial C_{Ni}}{\partial N_i})$$

$$(96)p_i^{*W} = c_{Li} + (C + F\frac{\partial C_{Ni}}{\partial N_i}) + \frac{\lambda}{1+\lambda}\frac{N_i}{\frac{\partial X_i}{\partial p_i}}(\frac{\partial X_i}{\partial N_i} - \frac{X_i}{N_i})$$

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As for the first-order condition with respect to frequency we note that the optimal price grows with the difference between the direct and the indirect demand effects of an increased supply, in this case in terms of more carriages.

### 4.3 Profit optimum

**Prices**

Since passenger groups do not affect each other the first-order condition with respect to price is the same as for the case where one passenger group was taken into account, that is:

$$(51)\ p^{*\pi} = c_L - \frac{X}{X_p} \equiv c_L - \frac{X(1 - X_G T_X)}{X_G} \equiv c_L - \frac{X}{X_G} + XT_X$$

**Frequency**

The first-order condition with respect to frequency yields:

$$(97)\frac{\partial \pi}{\partial F} = \Sigma_i p_i \frac{\partial X_i}{\partial F} - C - \Sigma_i c_{Li} \frac{\partial X_i}{\partial F} - f = 0$$

$$(98)\Sigma_i \frac{\partial X_i}{\partial F}(p_i - c_{Li}) = C + f$$

# 5 EXTERNAL EFFECTS AND SECOND-BEST PRICING

We will here investigate interactions between services that compete. One of these is a railway line. The other could be a railway line with rail but alternatively also an air or coach service or private car. The restriction to two services or modes does not reduce the generality of the analysis. The problem in focus is how optimal prices and frequencies change if one or both services do not charge optimal consumer prices or are not charged optimal infrastructure charges by a transport authority.

Since the interesting implications of taking into account two passenger groups and two classes were treated in the previous sections we will here assume one homogenous passenger group only.

We assume for simplicity that the two services have the same length in order to avoid a notation for distance. It is then implicitly understood that each departure has a specific distance. All parameters related to departure can then simply be converted to kilometres by use of the distance. For simplicity reasons we also ignore the number of carriages in a train since we here have competition between services or modes in focus.

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Section 5.1 deals with welfare optimum only, assuming possible deviations from optimal pricing. Section 5.2 deals with the case that both operators are profit maximising and consequently with second-best optima.

## 5.1 Welfare optimum

The welfare objective function for two competing services, e.g. rail and coach, rail and air, rail and rail, denoted k (=1 and 2) is:

$$(99)\ W = \sum_k S_k[G_k[p_1, p_2, F_1, F_2] + \\ +\sum_k p_k X_k[p_1, p_2, F_1, F_2] - \sum_k F_k C_k - \sum_k c_{Lk} X_k - \\ - \sum_k I_k F_k - \sum_k e_k F_k$$

We will now where sufficient confine ourselves to derivations with respect to one of the two services due to the symmetry.

The first-order condition with respect to the price of mode1 is:

$$(100)\frac{\partial W}{\partial p_1} = \frac{\partial S_1}{\partial G_1}\frac{\partial G_1}{\partial p_1} + \frac{\partial S_2}{\partial G_2}\frac{\partial G_2}{\partial p_1} + X_1 + p_1\frac{\partial X_1}{\partial p_1} + p_2\frac{\partial X_2}{\partial p_1} - c_{L1}\frac{\partial X_1}{\partial p_1} - c_{L2}\frac{\partial X_2}{\partial p_1} = 0$$

where

$$\frac{\partial G_1}{\partial p_1} \equiv 1 + \frac{\partial T_1}{\partial X_1}\frac{\partial X_1}{\partial p_1}$$

$$\frac{\partial G_2}{\partial p_1} \equiv \frac{\partial T_2}{\partial X_2}\frac{\partial X_2}{\partial p_1}$$

$$\frac{\partial S_k}{\partial G_k} \equiv - X_k$$

(100) can then be written as:

$$(101)\frac{\partial W}{\partial p_1} = (- X_1\frac{\partial T_1}{\partial X_1} + p_1 - c_{L1})\frac{\partial X_1}{\partial p_1} + (-X_2\frac{\partial T_2}{\partial X_2} + p_2 - c_{L2})\frac{\partial X_2}{\partial p_1} \equiv$$

$$\equiv \sum_k (- X_k\frac{\partial T_k}{\partial X_k} + p_k - c_{Lk})\frac{\partial X_k}{\partial p_1} = 0$$

where $M_{pi}$ and $M_{ci}$ reflect the marginal cost with respect to passengers and costs respectively.

If we denote the difference between price and social marginal consumer cost $d_i$ and express the differentials as $X^i_{pj}$, we can also write this condition in matrix form as:

$$(102)\begin{pmatrix} X^1_{p1} & X^2_{p1} \\ X^2_{p1} & X^2_{p2} \end{pmatrix}\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = 0$$

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In optimum of course the first-order condition with respect to prices requires that $d_1 = d_2 = 0$.

The first-order condition with respect to frequency is:

$$(103)\ \frac{\partial L}{\partial F_1} = \frac{\partial S_1}{\partial G_1}\frac{\partial G_1}{\partial F_1} + \frac{\partial S_2}{\partial G_2}\frac{\partial G_2}{\partial F_1} + p_1\frac{\partial X_1}{\partial F_1} + p_2\frac{\partial X_2}{\partial F_1} - C_1 - c_{L1}\frac{\partial X_1}{\partial F_1} - c_{L2}\frac{\partial X_2}{\partial F_1} - I_1 - e_1 = 0$$

where

$$\frac{\partial S_1}{\partial G_1}\frac{\partial G_1}{\partial F_1} = -X_1\frac{\partial T_1}{\partial X_1}\left(\frac{\partial X_1}{\partial F_1} - \frac{X_1}{F_1}\right) - X_1\frac{\partial T_1^{\tau}}{\partial F_1}$$

$$\frac{\partial S_2}{\partial G_2}\frac{\partial G_2}{\partial F_1} = -X_2\frac{\partial T_2}{\partial X_2}\frac{\partial X_2}{\partial F_1}$$

(103) can then be rewritten as:

$$(104)\ \sum_k\left(p_k - c_{Lk} - X_k\frac{\partial T_k}{\partial X_k}\right)\frac{\partial X_k}{\partial F_1} + X_1\frac{\partial T_1}{\partial X_1}\frac{X_1}{F_1} - X_1\frac{\partial T_1^{t}}{\partial F_1} - C_1 - I_1 - e_1 = 0$$

The first term reproduces the first-order condition with respect to prices. The second term reflects the direct marginal effect on in-vehicle congestion due to frequency increase. The third term reflects the positive external effect on frequency delay. The other terms reflect cost per departure, infrastructure cost and environmental cost.

If we express the differentials as $X^{i}_{Fj}$, we can write this condition in matrix form as:

$$(105)\ \begin{pmatrix} X^1_{F1} X^2_{F1} \\ X^1_{F2} X^2_{F2} \end{pmatrix}\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} -X_1\frac{\partial T_1}{\partial X_1}\frac{X_1}{F_1} + C_1 + I_1 + e_1 + X_1\frac{\partial T_1^{\tau}}{\partial F_1} \\ -X_2\frac{\partial T_2}{\partial X_2}\frac{X_2}{F_2} + C_2 + I_2 + e_2 + X_2\frac{\partial T_2^{\tau}}{\partial F_2} \end{pmatrix}$$

The corresponding expression for the difference between actual price and social marginal cost for each profit maximising operator is (see (72)):

$$(106a)\ X^1_{F1}d_1 = -X_1\frac{\partial T_1}{\partial X_1}\frac{\partial X_1}{\partial F_1} - s_1\frac{\partial X_1}{\partial F_1} + C_1 + f_1$$

$$(106b)\ X^2_{F2}d_2 = -X_2\frac{\partial T_2}{\partial X_2}\frac{\partial X_2}{\partial F_2} - s_2\frac{\partial X_2}{\partial F_2} + C_2 + f_2$$

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Since we want the welfare and the profit maximisation criteria to be fulfilled simultaneously, we can make use of the differences between the left-hand and the right-hand sides of the equation systems (105) and (106) to achieve:

$$(107a) -X_{F1}^{2}d_2 = f_1 - I_1 - e_1 - X_1\frac{\partial T_1}{\partial X_1}\left(\frac{\partial X_1}{\partial F_1}-\frac{X_1}{F_1}\right) - s_1\frac{\partial X_1}{\partial F_1} - X_1\frac{\partial T_1^t}{\partial F_1}$$

$$(107b) -X_{F2}^{1}d_1 = f_2 - I_2 - e_2 - X_2\frac{\partial T_2}{\partial X_2}\left(\frac{\partial X_2}{\partial F_2}-\frac{X_2}{F_2}\right) - s_2\frac{\partial X_2}{\partial F_2} - X_2\frac{\partial T_2^t}{\partial F_2}$$

$$(108) f_1 = I_1 + e_1 + X_1\frac{\partial T_1}{\partial X_1}\left(\frac{\partial X_1}{\partial F_1}-\frac{X_1}{F_1}\right) + X_1\frac{\partial T_1^t}{\partial F_1} + s_1\frac{\partial X_1}{\partial F_1} - \frac{\partial X_2}{\partial F_1}d_2 \equiv$$

$$\equiv I_1 + e_1 + X_1\frac{\frac{\partial X_1}{\partial F_1}}{\frac{\partial X_1}{\partial G_1}} + s_1\frac{\partial X_1}{\partial F_1} - \frac{\partial X_2}{\partial F_1}d_2$$

$$(109) s_1 = \frac{f_1 - I_1 - e_1 - X_1\frac{\partial T_1}{\partial X_1}\left(\frac{\partial X_1}{\partial F_1}-\frac{X_1}{F_1}\right) - X_1\frac{\partial T_1^t}{\partial F_1} + \frac{\partial X_2}{\partial F_1}d_2}{\frac{\partial X_1}{\partial F_1}} \equiv$$

$$\equiv \frac{f_1 - I_1 - e_1}{\frac{\partial X_1}{\partial F_1}} - \frac{X_1}{\frac{\partial X_1}{\partial G_1}} + \frac{\frac{\partial X_2}{\partial F_1}}{\frac{\partial X_1}{\partial F_1}}d_2$$

Assume now that $d_2$ is below its first-best optimal value, due to that $f_2$ is lower than the optimal value and/or that $s_2$ is higher than the optimal value. One reason may be that the authority reduces the charge for the infrastructure and environmental costs at a larger extent than what is motivated by the positive external effects $(X/X_G)X_F$.

It is then clear that $d_1$ has to be below its first-best optimal value. This situation is obtained if $f_1$ is lower than the first-best optimal value and/or that $s_1$ is higher than the first-best optimal value.

The decrease of $f_1$ and/or increase of $s_1$ are larger the larger are the cross-elasticities between the competing lines or modes, the larger are $dX_2/dF_1$ and $dX_1/dF_2$. The deviations of $f_1$ and $s_1$ from their first-best values, $f_1^*$ and $s_1^*$, can also be expressed explicitly by using each of these first-best values given that either $s_1=0$ and $f_1^* = I_1+e_1 + X_1(dX_1/dF_1)/(dX_1/dG_1)$, or $f_1= I_1+e_1$ $s_1^* = - X_1/(dX_1/dG_1)$. We then achieve the following additional reduction of the first-best optimal infrastructure charge and the additional subsidy of the consumer price respectively:

$$(110)\, f_1 - f_1^* = -\frac{\partial X_2}{\partial F_1}d_2$$

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$$(111)\quad s_1 - s_1^* = \frac{\frac{\partial X_2}{\partial F_1}}{\frac{\partial X_1}{\partial F_1}} d_2$$

If $d_2=0$ we notice that the infrastructure charge and the subsidy are exactly those that were obtained for corrections in optimum, according to expressions (11) – (15).

If $d_2<0$, for example due to that line/mode 2 has a too low infrastructure charge we notice that the infrastructure charge of line/mode 1 must be reduced and/or the subsidy of mode 1 be increased compared to the situation where optimal corrections are employed.

If $d_2>0$, for example due to that line/mode has a too high infrastructure charge and/or the politicians cannot or will not correct for monopolistic pricing, the infrastructure charge of mode 1 should be increased and/or the subsidy be reduced, compared to the situation where optimal corrections are employed.

The corrections of subsidies and infrastructure charge grow with the cross-elasticity with respect to frequency.

The larger the own elasticity with respect to frequency is, the smaller the subsidy can be; the subsidy "bites better" in this case.

Whether corrections ought to be made through subsidies or infrastructure charges is a practical matter. This issue in turn is dependent on the organisation of the transport system in general and whether the railway is vertically integrated or separated.

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# 6 SUMMARY OF SIMULATION WORK

## 6.1 Introduction

The theoretical analysis demonstrated that railway services ought not to cover the full operation costs. If competing modes such as air services do not fully cover external costs this forms an additional argument for subsidising the railway.

Here we simulate taking away the base boarding fare and reduce the kilometre charge by 30% for 2nd class passengers, on routes with low capacity use and few carriages.

This section briefly summarises the results that were found by simulation of rail fare reductions, assumed to come true through subsidies related to the passengers. The simulations were made by use of the VIPS assignment system. This system simultaneously distributes passengers between routes within each public transport mode and between public transport modes and car. It also calculates revenues and costs per route and the passengers' costs in terms of price and travel time components. The system thereby takes into account time and distance of each link in the network and the price of each mode per kilometre, progressive or regressive. Each mode was also given a specific value of time in order to reflect the comfort of the mode.

Since different passenger groups meet different prices for various modes, have different values of ride time and wait time and have different availability of car, the demand was separated into groups that were also analysed separately. The table below shows the segmentation and the assumed values of ride time and the weights for each travel time component in relation to ride time.

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**Table 6.1.1 Segmentation of demand, assumed time values and weights**

| **Ride time value and weights for the other components** | Share % | Value of time (IC-train) SEK/hour | Wait time weight | Transfer time weight | Walk time weight | Transfer pemalty minutes |
|---|---|---|---|---|---|---|
| **Private journeys** | | | | | | |
| *Long distance, >100 km* | 100 | | | | | |
| Working | 32 | 100 | 0,6 | 2,0 | 2,0 | 10 |
| Working, no car available | 3 | 60 | 0,5 | 2,0 | 2,0 | 10 |
| Working, car available | 29 | 60 | 0,5 | 2,0 | 2,0 | 10 |
| Studying, no car available | 4 | 35 | 0,4 | 2,0 | 2,0 | 10 |
| Studying, car available | 18 | 35 | 0,4 | 2,0 | 2,0 | 10 |
| Pensioner, no car available | 3 | 35 | 0,3 | 2,0 | 2,5 | 10 |
| Pensioner, car available | 11 | 35 | 0,3 | 2,0 | 2,5 | 10 |
| *short distande, <100 km* | 100 | 20 | 0,8 | 2,0 | 2,0 | 5 |
| **Business journeys** | | | | | | |
| Business, low value of time | 50 | 200 | 1,2 | 2,0 | 2,0 | 20 |
| Business, high value of time | 50 | 400 | 1,2 | 2,0 | 2,0 | 20 |

| **Ride time weights** | Share % | IC-train | Night train | X2000 High-speed | Coach long distance | Bus regional | Air | Car |
|---|---|---|---|---|---|---|---|---|
| **Private journeys** | | | | | | | | |
| *Long distance, >100 km* | 100 | | | | | | | |
| Working | 32 | 1,0 | 0,45 | 0,9 | 1,3 | 1,4 | 1,1 | 2,0 |
| Working, no car available | 3 | 1,0 | 0,45 | 0,9 | 1,3 | 1,4 | 1,1 | 1,8 |
| Working, car available | 29 | 1,0 | 0,45 | 0,9 | 1,3 | 1,4 | 1,1 | 1,8 |
| Studying, no car available | 4 | 1,0 | 0,45 | 0,9 | 1,2 | 1,2 | 1,1 | 2,0 |
| Studying, car available | 18 | 1,0 | 0,45 | 0,9 | 1,2 | 1,2 | 1,1 | 2,0 |
| Pensioner, no car available | 3 | 1,0 | 0,45 | 0,9 | 1,0 | 1,2 | 1,1 | 1,4 |
| Pensioner, car available | 11 | 1,0 | 0,45 | 0,9 | 1,0 | 1,2 | 1,1 | 1,4 |
| *short distande, <100 km* | 100 | 1,0 | 0,45 | 0,9 | 1,5 | 1,2 | 1,1 | – |
| **Business journeys** | | | | | | | | |
| Business, low value of time | 50 | 1,0 | 0,45 | 0,9 | 1,3 | 1,4 | 1,1 | 2,0 |
| Business, high value of time | 50 | 1,0 | 0,45 | 0,9 | 2,5 | 1,5 | 1,1 | 2,0 |

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## 6.2 Results

### 6.2.1 Shift of modes

The reduced prices of certain railway lines would of course change the mode split, shown in the table below. Since the lines with reduced prices comprise rather small ones the total effects on the whole network are not large, but of course demand for train increases and are reduced for competing modes.

**Tabell 6.2.1.1 Calculated demand in millions of boardings, passenger kilometres and shares**

| | Base network | | | Alternative with price reductions | | | Difference | |
|---|---|---|---|---|---|---|---|---|
| | **Boarding millions** | **Pass. Km millions** | **Share % pkm** | **Boarding millions** | **Pass. Km millions** | **Share % pkm** | **Pass. Km millions** | **%** |
| Train | 99,4 | 7517 | 50 | 100,6 | 7655 | 50 | 138 | 2 |
| *of which X2000* | *5,3* | *2077* | *14* | *5,4* | *2090* | *14* | *13* | *1* |
| *of which Night train* | *1,1* | *928* | *6* | *1,1* | *938* | *6* | *10* | *1* |
| *of which other trains* | *92,9* | *4511* | *30* | *94,1* | *4627* | *30* | *115* | *3* |
| Air | 7,7 | 3625 | 24 | 7,7 | 3619 | 24 | -6 | 0 |
| *of which SAS* | *5,8* | *2841* | *19* | *5,8* | *2839* | *19* | *-3* | *0* |
| *of which other airlines* | *1,9* | *784* | *5* | *1,9* | *780* | *5* | *-4* | *0* |
| Coach | 4,9 | 1251 | 8 | 4,7 | 1218 | 8 | -33 | -3 |
| Regional bus etc. | 75,7 | 2783 | 18 | 75,5 | 2766 | 18 | -17 | -1 |
| **Total public transport** | **187,7** | **15176** | **100** | **188,4** | **15257** | **100** | **82** | **1** |
| Car | 92,4 | 23398 | 61 | 92,1 | 23322 | 60 | -77 | 0 |

The next table shows the capacity, average load and average capacity use for the base situation and for reduced prices for each of the lines that were price reduced.

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**Table 6.2.1.2 Capacity use**

| | | | | Base network | | | Alt. P-30% | | |
|---|---|---|---|---|---|---|---|---|---|
| **Route** | **Alignment** | **No. Of carriages** | **Capacity** | **Average load>>** | **Average last<<** | **Load/ cap.** | **Average load>>** | **Average last<<** | **Load/ cap.** |
| 30a | tLuleå=tRiksgräns | 5 | 219 000 | 7 066 | 12 112 | 0,04 | 20 369 | 18 610 | 0,09 |
| 30b | tKiruna=tRiksgräns | 3 | 65 700 | 0 | 3 | 0,00 | 0 | 3 | 0,00 |
| 41b | tStockhlmC=tGävle | 5 | 593 898 | 83 152 | 77 614 | 0,14 | 148 987 | 141 902 | 0,24 |
| 41h | Sundsvall=Långsele | 2 | 31 286 | 16 674 | 7 895 | 0,39 | 17 550 | 8 321 | 0,41 |
| 48LT | Borlänge=Malung | 2 | 175 200 | 70 233 | 49 805 | 0,34 | 81 867 | 57 644 | 0,40 |
| 49b | tStockhlmC=tFalun | 5 | 328 500 | 53 646 | 52 727 | 0,16 | 92 526 | 105 651 | 0,30 |
| 49e | tAvesta=tLudvika | 2 | 112 729 | 17 570 | 16 062 | 0,15 | 18 391 | 17 174 | 0,16 |
| 53f | tÖrebro=tMjölby | 3 | 274 226 | 28 535 | 30 740 | 0,11 | 52 475 | 36 535 | 0,16 |
| 55bLT | tVästerås=tFagerstaN | 2 | 512 780 | 141 582 | 145 725 | 0,28 | 154 213 | 160 209 | 0,31 |
| 55cLT | tVästerås=tRamnäs br | 2 | 106 451 | 28 569 | 13 678 | 0,20 | 29 730 | 13 789 | 0,20 |
| 56EsFl | tEskilstun=tFlen | 3 | 140 786 | 12 298 | 11 419 | 0,08 | 12 546 | 11 437 | 0,09 |
| 56KaNo | tKatrineho=tNorrköpin | 3 | 93 857 | 10 156 | 9 944 | 0,11 | 11 459 | 10 549 | 0,12 |
| 56SaVäLT | tSala=tVästerås | 3 | 663 916 | 137 649 | 135 206 | 0,21 | 158 811 | 161 457 | 0,24 |
| 56VäEsLT | tVästerås=tEskilstun | 3 | 281 571 | 77 779 | 72 108 | 0,27 | 108 564 | 99 187 | 0,37 |
| 57ArVäLT | tArboga=tVästerås | 2 | 125 143 | 7 840 | 16 441 | 0,10 | 12 935 | 31 469 | 0,18 |
| 57fLT | tVästerås=tKöping | 2 | 93 857 | 36 334 | 41 144 | 0,41 | 37 123 | 43 217 | 0,43 |
| 58a | tStockhlmC=tEskilstun | 6 | 338 188 | 55 781 | 61 736 | 0,17 | 76 959 | 85 465 | 0,24 |
| 58d | tHallsberg=tEskilstun | 2 | 56 289 | 8 817 | 9 043 | 0,16 | 9 505 | 10 096 | 0,17 |
| 60g | tSkövde=tGöteborg | 5 | 422 169 | 34 073 | 37 261 | 0,08 | 45 551 | 49 778 | 0,11 |
| 63aLT | tHallsberg=tHerrljung | 2 | 153 300 | 29 753 | 40 147 | 0,23 | 40 800 | 53 999 | 0,31 |
| 63bLT | tLidköping=tHerrljung | 2 | 160 839 | 20 133 | 24 466 | 0,14 | 22 969 | 28 130 | 0,16 |
| 63cLT | tMariestad=tHerrljung | 2 | 73 000 | 3 799 | 4 789 | 0,06 | 4 215 | 5 203 | 0,06 |
| 63dLT | tHallsberg=tLidköping | 2 | 87 600 | 9 194 | 9 014 | 0,10 | 13 099 | 10 755 | 0,14 |
| 65a | tFalköping=tNässjö | 4 | 516 093 | 36 212 | 47 178 | 0,08 | 51 470 | 58 491 | 0,11 |
| 65bLT | tJönköping=tNässjö | 4 | 396 286 | 56 069 | 62 937 | 0,15 | 73 175 | 89 064 | 0,20 |
| 70dLT | tKarlstad=tCharlotte | 4 | 573 931 | 47 309 | 61 626 | 0,09 | 55 836 | 74 323 | 0,11 |
| 70fLT | tKarlstad=tArvika | 2 | 37 543 | 6 346 | 4 487 | 0,14 | 7 074 | 4 862 | 0,16 |
| 71c | tGöteborg=tÅmål | 4 | 62 571 | 5 634 | 5 249 | 0,09 | 10 639 | 8 583 | 0,15 |
| 73bLT | tKristineh=tKarlstad | 4 | 554 800 | 61 167 | 59 260 | 0,11 | 71 157 | 76 258 | 0,13 |
| 73c | tLaxå=tKristineh | 2 | 93 857 | 3 710 | 3 906 | 0,04 | 4 017 | 4 832 | 0,05 |
| 73dLT | tKarlstad=tKil | 2 | 288 000 | 13 592 | 12 765 | 0,05 | 14 746 | 13 068 | 0,05 |
| 74aLT | tKarlstad=tTorsby | 2 | 144 000 | 43 895 | 46 144 | 0,31 | 48 009 | 50 035 | 0,34 |
| 74bLT | tKarlstad=tSunne | 2 | 56 365 | 7 879 | 8 199 | 0,14 | 8 886 | 8 941 | 0,16 |
| 81b | tStockhlmC=tLinköping | 4 | 237 559 | 40 792 | 37 748 | 0,17 | 55 375 | 49 838 | 0,22 |
| 81c | tNyköping=tNorrköpin | 2 | 181 241 | 24 236 | 25 554 | 0,14 | 24 979 | 25 835 | 0,14 |
| 81d | tLinköping=tMjölby | 2 | 93 857 | 13 395 | 11 105 | 0,13 | 13 438 | 11 144 | 0,13 |
| 81iLT | tTranås=tNässjö | 2 | 336 000 | 80 217 | 63 736 | 0,21 | 107 571 | 74 531 | 0,27 |
| 83 | tLinköping=tVästervik | 2 | 269 538 | 47 416 | 50 381 | 0,18 | 48 896 | 51 246 | 0,19 |
| 84b | tLinköping=tKisa | 4 | 62 571 | 2 069 | 2 444 | 0,04 | 2 087 | 2 444 | 0,04 |
| 84c | tLinköping=tKalmar | 2 | 138 576 | 16 381 | 9 450 | 0,09 | 23 666 | 17 610 | 0,15 |
| 85aLT | tNässjö=tHultsfred | 2 | 233 600 | 51 575 | 30 681 | 0,18 | 76 356 | 58 459 | 0,29 |
| 85b | Berga=Oskarshamn | 2 | 240 471 | 44 254 | 2 540 | 0,10 | 44 481 | 2 838 | 0,10 |
| 86aLT | tNässjö=tHalmstad | 4 | 416 100 | 92 809 | 113 466 | 0,25 | 145 983 | 156 551 | 0,36 |
| 87bLT | tJönköping=tVaggeryd | 2 | 87 600 | 26 813 | 17 871 | 0,26 | 28 556 | 18 982 | 0,27 |
| 88aLT | tNässjö=tÅseda | 2 | 94 886 | 16 564 | 13 502 | 0,16 | 21 290 | 15 922 | 0,20 |
| 88bLT | tNässjö=tVetlanda | 2 | 226 065 | 54 942 | 47 401 | 0,23 | 56 966 | 51 998 | 0,24 |
| 89LT | tNässjö=tSävsjö | 2 | 182 364 | 44 009 | 44 646 | 0,24 | 46 085 | 47 100 | 0,26 |
| 90a | Karlskrona=Malmö | 2 | 533 217 | 203 899 | 206 603 | 0,38 | 214 209 | 219 677 | 0,41 |
| 92a | tHelsingbo=tKristians | 3 | 187 714 | 33 732 | 23 713 | 0,15 | 40 061 | 27 735 | 0,18 |
| 94 | tMalmö=tTrellebor | 3 | 131 400 | 19 512 | 22 759 | 0,16 | 19 512 | 22 759 | 0,16 |
| 95b | tAlvesta=tKalmar | 4 | 412 235 | 56 133 | 71 441 | 0,15 | 65 735 | 79 438 | 0,18 |
| 96 | tEmmaboda=tKarlskron | 1 | 266 609 | 27 300 | 35 214 | 0,12 | 32 007 | 39 812 | 0,13 |
| 98LT | tBorås=tVarberg | 2 | 339 097 | 120 310 | 164 640 | 0,42 | 136 786 | 188 281 | 0,48 |
| 100c | tGöteborg=tHelsingbo | 4 | 87 600 | 14 789 | 14 823 | 0,17 | 20 857 | 21 822 | 0,24 |
| 100d | tHalmstad=tMalmö | 4 | 118 886 | 11 810 | 10 261 | 0,09 | 14 507 | 11 934 | 0,11 |
| 100e | tEd=tGöteborg | 4 | 250 286 | 17 517 | 18 305 | 0,07 | 17 766 | 19 158 | 0,07 |
| 100f | tGöteborg=tHalmstad | 4 | 257 547 | 26 023 | 29 382 | 0,11 | 31 482 | 37 999 | 0,13 |
| 107aLT | tMalmö=tYstad | 2 | 750 857 | 326 407 | 320 829 | 0,43 | 333 993 | 329 864 | 0,44 |
| 130aLT | Göteborg=Strömstad | 2 | 204 400 | 101 327 | 68 809 | 0,42 | 125 566 | 110 087 | 0,58 |
| 130bLT | tGöteborg=tUddevalla | 2 | 190 140 | 101 694 | 54 938 | 0,41 | 110 744 | 66 563 | 0,47 |

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### 6.2.2 The passengers

Table 6.2.2.1 below shows the change of generalised cost per journey and the change of consumer surplus for each group of passengers. Since the price reductions refer to few lines the effects per journey are small on the national level.

| | **Business 300 SEK/h** | **Working 100 SEK/h** | **Working 60 SEK/h** | **Pens 35 SEK/h** | **Stud 20 SEK/h** | **Trips< 100 km 36 SEK/h** | **Sum** |
|---|---|---|---|---|---|---|---|
| Gen. cost, SEK/journey | 0,10 | 0,40 | 1,09 | 2,19 | 1,49 | 0,00 | 5,27 |
| Gen. cost, %/journey | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,01 |
| Cons. surplus MSEK/yea | 3 | 12 | 32 | 28 | 30 | 0 | 103 |
| *of which fare* | *2* | *16* | *36* | *29* | *33* | *0* | *116* |
| *of which time* | *0* | *-4* | *-5* | *-1* | *-3* | *0* | *-13* |

The largest gains are made by pensioners and students. Most groups lose time since many change from air to the cheaper but slower train. Business travellers with the lower value of time also gain time, which may depend on that some shift from coach and car.

### 6.2.3 The operators

The table below shows the calculated changes of revenues and costs for various operators and modes. Evidently the railway is supposed to increase its losses for the already unprofitable lines, and the competitors lose as well. No concern is then taken to that air and coach operators may adjust their supply in terms of frequency or vehicle size, something thta would reduce thier losses.

**Table 6.2.3.1 Calculated change of revenues and costs per operator and mode in millions SEK per year**

| | **Revenues, MSEK** | | | **Costs** | **Revenues-** |
|---|---|---|---|---|---|
| | Business | Private | Sum | **MSEK** | **costs, MSEK** |
| Train | 0 | -12 | -12 | 14 | -26 |
| *of which X2000* | *2* | *10* | *12* | *0* | *12* |
| *of which Night train* | *0* | *5* | *5* | *0* | *5* |
| *of which other trains* | *-2* | *-26* | *-29* | *0* | *-29* |
| Air | -1 | -8 | -9 | -1 | -9 |
| *of which SAS* | *0* | *-3* | *-3* | *0* | *-3* |
| *of which other airlines* | *-1* | *-5* | *-6* | *0* | *-6* |
| Coach | 0 | -16 | -16 | -3 | -13 |
| Regional bus etc. | 0 | -12 | -12 | 0 | -12 |
| **Total** | **-1** | **-48** | **-50** | **10** | **-59** |

### 6.2.4 Welfare

The table below summarises all components in the cost-benefit analysis.

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| Benefits and costs | MSEK/year |
|---|---|
| Consumer surplus | 103 |
| *of which time* | *116* |
| *of which price* | *-13* |
| Private sector finances | -59 |
| Cost adjustment | -2 |
| Net public surplus | -30 |
| Excess burden | -9 |
| External effects | 17 |
| **Sum** | **20** |

The price reductions assumed to be due to subsidies seem to generate a small social net benefit. Even if this benefit is small it demonstrates that subsidies may be an alternative to the current procurement of non-profitable railway lines. One should also consider that the losses of air anc coach operators may be overestimated since no concern has been taken to possible adjustments.

# 7 CONCLUSIONS

The theoretical part of the work shows in short that:

- Welfare optimum implies a price below average operating cost while profit optimum implies a price above average operating cost. The low welfare oriented price is due to concern for the passengers' benefits of increased frequency, which affects both ride time and wait time costs. Welfare optimum also implies a higher level of frequency than profit optimum, at least if the number of carriages is the same in the two cases. And welfare optimum implies a higher number of carriages than profit optimum, at least if the frequency is the same in the two cases. The non-optimal behaviour of the profit maximising operator can be corrected either by use of a subsidy related to the ticket price or a reduction of the infrastructure charge or by a combination of the two.

- If there is a binding budget constraint the findings are as follows. Assume that 1$^{st}$ class passengers have a high value of wait time and ride time costs and that 2$^{nd}$ class passengers have relatively low values. Assume also that 1$^{st}$ class passengers have low price elasticity and that 2$^{nd}$ class passengers have relatively high price elasticity. Then the contribution to cost coverage from 1$^{st}$ class passengers grows with respect to the low price elasticity. This is the ordinary Ramsey-pricing rule. But the contribution from 1$^{st}$ class passengers is reduced with respect to the valuations of ride time and wait time. It is thus an empirical issue whether 1$^{st}$ or 2$^{nd}$ class passengers would contribute most to cost coverage in optimum when no distribution concern is taken.

- In a second-best situation the corrections mentioned have to be modified. If for example one mode pays a too low infrastructure charge the competing modes should be subsidised either by a higher subsidy related to the ticket and/or a higher reduction of the infrastructure charge.

The simulation part of the work backs up the theoretical part by the finding that a subsidy that leads to a reduction of the consumer price may imply a net social benefit.

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